1. The original economy has an endogenous saving rate. The saving rate is lower at \( s_1 \) when income per capita \( y \) is below \( \bar{y} \), and the saving rate is higher at \( s_2 \) when \( y \) is above \( \bar{y} \). The diagram is show on a separate page. For this economy, there are two possible steady states. The lower steady state happens at \( k_1, y_1 \), where 
\[
zs_1 f(k) = (n + d)k
\]
, and the other steady state is at \( k_2, y_2 \), where 
\[
zs_2 f(k) = (n + d)k
\]
. The lower steady state is frequently referred to as a poverty "trap". There are two ways to explain why this is called a trap. The first way is to note that if this economy receives, say foreign aid, but the foreign aid is not sufficient to lift this economy's income per capita beyond \( \bar{y} \), then this economy is doomed to remain at the lower steady state forever. The second way is to make use of the assumption of an endogenous saving rate. For example, this economy can escape the poverty trap if its saving rate increases. But saving rate cannot increase unless \( y \) increases, but \( y \) cannot increase unless \( s \) increases. Thus, this economy is trapped in a viscous cycle of poverty.

Economy B is identical to the original economy but with only one saving rate of \( s_2 \). The steady state of economy B is at \( k_2, y_2 \), where 
\[
zs_2 f(k) = (n + d)k
\]
. Starting when both economies are at steady \( y^* > \bar{y} \), which is at \( k_2, y_2 \), where 
\[
zs_2 f(k) = (n + d)k
\]
. When half of the capital stock is destroyed, the steady states for both economies are not changed since the parameters that determine the steady states \( (s_1, s_2, n, d, z) \) are not affected by this one time event. For economy B, it will temporary move to a point left of the steady state and below \( y \). But, over time, Economy B will return to steady state at \( k_2, y_2 \). The original economy is not so fortunate. Since \( y \) is now less than \( \bar{y} \), it will now converge to the lower steady state at \( k_1, y_1 \).

2. For this problem, let's start with the conventional utility maximization problem with only the lifetime budget constraint. Let 
\[
we = (y - t) + \frac{y_t - t}{1 + r}
\]
. For the conventional case, a borrower can choose to borrow an amount \( (x^*) \) such that 
\[
(y - t) + x^* \leq we
\]
. The amount of borrowing \( (x^*) \) is chosen to maximize the borrower's utility. If there is now a borrowing constraint, where \( (x) \) is the maximum that could be borrowed, borrowers now face an additional constraint that 
\[
(y - t) + x < we,
\]
rewriting, 
\[
x < (y - t) + \frac{y_t - t}{1 + r} - (y - t),
\]
or 
\[
x < \frac{y_t - t}{1 + r}
\]
as stated in the problem. There are now two types of borrowers. The first type is the non-binding constraint borrowers. These are borrowers that choose to borrow \( x^* \leq x \), shown in Figure 2(a) for the case where \( x^* = x \), i.e., the borrowers maximize utility by borrowing exactly the amount \( x \). The constraint is not binding because the amount of borrowing that maximizes utility is not affected if more could be borrowed. The second type of borrowers face a binding borrowing constraint, this is the case where \( x^* > x \). Figure 2(b) shows such a case. This borrower would like to borrow sufficiently to reach indifference curve \( I_1 \), but because of the borrowing constraint, can only borrow \( x \) to reach indifferent curve \( I_2 \).

Now there is an increase in the borrowing limit so that \( x_1 > x \). For the non-binding constraint borrowers, this does not affect the original equilibrium (recall that for this group of borrowers, the original borrowing amount maximizes their utility). So, nothing changes. For the binding constrained borrowers, however, they can reach a higher indifference curve \( I_2 \) by borrowing the larger amount. Thus, for this group, borrowing increases (savings decreases), current consumption increases, but future consumption must decrease (they still have to meet the life-time budget constraint).
3. The results of this problem are the same for both borrowers and savers, except an increase in savings for the savers is the same as a decrease in borrowing for the borrowers. It is, however, easier to show the case for a saver, which is shown in Figure 3. The original equilibrium is at \((c_1, c_1')\) and savings is given by the segment \(S_1\) on the horizontal axis. After a permanent increase in income with no change in \(t\) and \(t'\), the final equilibrium is at \((c_2, c_2')\), with an increase in both \(c\) and \(c'\) since both are normal goods. Savings, as stated, could increase, remain the same, or decrease. You are asked to show in the diagram the case where savings decreases. The savings after a permanent increase in income is given by the segment \(S_2\) which can be seen is less than \(S_1\).

To explain the indeterminate result for savings, we can analyze the question in two steps. First, all else equal, an increase in \(y'\) will increase \(c\) and \(c'\) since they are normal goods. This implies that \(\frac{dc}{dy'} > 0\). Now \(s = y - t - c\), divide by \(dy'\), and noting that \(dy = dt = 0\), we obtain \(\frac{ds}{dy'} = -\frac{dc}{dy'} < 0\). Next, repeat the same experiment, except that this time increase \(y\) and holding all else constant. Again, \(c\) and \(c'\) will increase since they are normal goods. This implies that \(\frac{dc}{dy} > 0\). Again, \(s = y - t - c\), divide by \(dy\), and noting that this time only \(dt = 0\), we obtain \(\frac{ds}{dy} = 1 - \frac{dc}{dy}\). Now for \(\frac{ds}{dy} > 0\), the assumption of consumption smoothing is necessary, which we'll make here. Now \(\frac{ds}{dy'} < 0\) and \(\frac{ds}{dy} > 0\) which imply the net effect is indeterminate.

Using the hints that are given, first demonstrate that:

(i) \(\frac{ds}{dy} = 1 - \frac{dc}{dy} > 0\). This was done partially above, we need to show only that \(1 - \frac{dc}{dy} > 0\). First, note that

\[ |D| = -U_{11} + (1 + r)U_{12} + (1 + r)[U_{12} - (1 + r)U_{22}] > 0 \]

is given. Using the first-order conditions that are given and Cramer's rule, we find that \(\frac{dc}{dy} = \frac{(1 + r)(U_{12} - (1 + r)U_{22})}{|D|} > 0\), which is what we needed. Next we need to show that \((1 - \frac{dc}{dy'}) > 0\).

(ii) Now we have already demonstrated that \(\frac{ds}{dy'} = -\frac{dc}{dy'} < 0\) above, we need only to demonstrate that \(\frac{dc}{dy'} > 0\).

Again using the first-order conditions that are given and Cramer's rule, we find that

\[ \frac{dc}{dy'} = \frac{U_{12} - (1 + r)(U_{22})}{|D|} = \frac{1}{(1 + r)} \frac{dc}{dy'} > 0. \]
Figure (3)