The reduction in the capital stock shifts the production function down from $ZF(K, N)_1$ to $ZF(K, N)_2$. Marginal product of labor is reduced at each labor input level. Thus, labor demand shifts down from $N_1^{ed}$ to $N_2^{ed}$.

Employment increases from $N_1$ to $N_2$.

Real wage increases from $w_1$ to $w_2$. Depending on the increase in labor input, output can increase, decrease, or remain the same.

(i) decrease (at $N_2^{**}$), output supply curve shifts left.
(ii) remain the same ($N_2^{**}$), output supply curve does not change.
(iii) increase (at $N_2^{***}$), output supply curve shifts to the right.

On the output demand side, the reduction in the stock of Capital means the marginal product of capital increases next period increases. Thus, investment increases, shifting $Y_t$ out to the right.
Three possibilities,

(i) Output supply curve shifts left.

(ii) Output supply curve shifts right.

(iii) Output supply curve does not change.

(i) Output supply curve shifts left (decrease):

At low output equilibrium, equilibrium shifts from $L_1$ to $L_2$. Equilibrium output falls, real interest rate rises, or remain the same.

At high output equilibrium, equilibrium changes from $H_1$ to $H_2$. Equilibrium output increases, real interest rate falls.

(ii) Output supply curve shifts right (increase):

At low output equilibrium, equilibrium shifts from $L_1$ to $L_2$. Equilibrium output rises, real interest rate can rise, fall or stay the same.

At high output equilibrium, $H_1$ to $H_2$. Equilibrium real interest rate rises. Equilibrium output, rise, fall, stay the same?
(iii) No change in output supply.

At low output equilibrium, equilibrium shifts from $L_1$ to $L_2$. Both $r$ and $Y$ increase.

At high output equilibrium, equilibrium changes from $H_1$ to $H_2$. $Y$ increases, $r$ decreases.
(a) Budget constraint when young:

\[ C_{t,t} + \frac{V_{t}}{V_{t+1}} M_{t} = y \quad \text{or simply} \quad C_{t} + \frac{V_{t}}{V_{t+1}} M_{t} = y \]

Budget constraint when old:

\[ C_{t,t+1} = \frac{V_{t}}{V_{t+1}} M_{t} \]

Lifetime budget constraint:

\[ C_{t,t} + \left[ \frac{V_{t}}{V_{t+1}} \right] C_{t,t+1} = y \]

Let \( q_{t} = \frac{V_{t}}{V_{t+1}} M_{t} \) - demand for money

We can rewrite:

\[ C_{t,t} = y - q_{t}, \quad C_{t,t+1} = \left[ \frac{V_{t+1}}{V_{t}} \right] q_{t} \]

(b) \( \max_{t} \quad U = E_{t} [ \ln (C_{t,t}) + \beta \ln (C_{t,t+1}) ] \)

Subject to:

\[ C_{t,t} + \left[ \frac{V_{t}}{V_{t+1}} \right] C_{t,t+1} = y \]

Can be rewritten as:

\[ \max_{t} \quad U = \ln (y - q_{t}) + \beta \ln \left( \frac{V_{t+1}}{V_{t}} \right) q_{t} \]

\[- \frac{1}{(y - q_{t})} + \frac{\beta}{(\frac{V_{t+1}}{V_{t}}) q_{t}} = 0 \]

\[ \beta (y - q_{t}) = q_{t}, \quad q_{t}^{*} = \frac{\beta y}{1 + \beta} \]
\( C_{t+1}^* = y/q_t^* = y - \frac{\beta y}{1+\beta} = \frac{y}{1+\beta} \)

\( C_{t+1}^* = \frac{v_{t+1}}{v_t} \frac{q_t^*}{q_t} = \frac{v_{t+1}}{v_t} \left[ \frac{\beta q_t}{1+\beta} \right] \)

\( \beta \) is preference for future consumption. The larger the \( \beta \), the stronger the preference for future consumption over current consumption.

An increase in \( \beta \), for example, decreases \( C_{t+1}^* \), and \( q_t^* \) increases.
a. Feasible set: \(100c_{1,t} + 100c_{2,t} \leq 100y = 100(20) \Rightarrow c_{1,t} + c_{2,t} \leq 20\)

The graph is easy. Horizontal and vertical intercepts equal 20. Note that until you know more about preferences, you cannot explicitly find \(c_1\) and \(c_2\), but you can draw a general graph.

b. First period: \(c_{1,t} + v_t m_t \leq y\)

Second period: \(c_{2,t+1} \leq v_{t+1} m_t\)

Lifetime: \(c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right]c_{2,t+1} \leq y \Rightarrow c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right]c_{2,t+1} \leq 20\)

c. The money market clearing condition is:

\[v_t M_t = N_t(y - c_{1,t}) \Rightarrow 400v_t = 100(y - c_{1,t})\]

\[\Rightarrow v_t = \frac{100(y - c_{1,t})}{400}\]
You could substitute for $y$ here, but this form is good enough for our purposes. We want to find $v_{t+1}/v_t$.

$$
\frac{v_{t+1}}{v_t} = \frac{\frac{100(y - c_{1,t+1})}{400}}{\frac{100(y - c_{1,t})}{400}} = 1.
$$

where the last equality follows from cancellation and imposing stationarity ($c_{1,t} = c_{1,t+1}$ for all $t$).

d. Since the rate of return on fiat money is one, we find that the real demand for fiat money is

$$
v_t m_t = \frac{y}{1 + \frac{v_t}{v_{t+1}}} = \frac{20}{1 + 1} = 10.
$$

Note from the first-period budget constraint that $v_t m_t$ is equal to $y - c_1$, so that $y - c_1 = 10$. Since $v_t m_t = 10$, $c_1 = y - v_t m_t = 20 - 10 = 10$. Half of the endowment is consumed and half is sold for real money balances. (Question to answer on your own: What will $c_2$ be?) Using $y - c_1$ in the expression derived in part c,

$$
v_t = \frac{100(10)}{400} = 2.5 \Rightarrow p_t = \frac{1}{v_t} = 0.4
$$

e. We saw in this chapter that the rate of return on fiat money is $n$ in an economy with a constant fiat money stock and a changing population. So an increase in $n$ will cause an increase in the rate of return on fiat money.

An increase in the rate of return on fiat money will increase real money balances. This should make intuitive sense and should be easy to see by plugging a few numbers into the money demand function (e.g., suppose that $v_{t+1}/v_t$ increases to 2; re-solve for real money demand).

Given that the real demand for fiat money increases, the money market-clearing condition tells us that value of money will increase in the initial period. This also should be intuitive—an increase in the demand for apples increases the value of apples.

Since the value of money increases in the initial period, the initial old (the initial holders of money) are made better off. (The initial old are better off whenever the real value of money in period 1 increases.)

f. Following part c, we get $v_t = \frac{100(10)}{800} = 1.25$. The value of money is cut in half (the price level doubles to $1/v_t = 0.8$). However, the rate of return on
fist money is still one if the population is held constant. Notice that the total real value of the fist money stock, which is initially held by the initial old) does not change (v is cut in half whereas M doubles). This implies that the welfare of the initial old does not change. Their holdings of money will not buy any more (or less) of the consumption good.

Exercise 4.8

a. Since N and M are constant, the rate of return on fist money in a stationary equilibrium will be one in each country. Intuitively, the economies are identical in the sense of how they change over time. They do not change. Even if each country has a different value of money, v, that value does not change over time; therefore, the rate of return v_{t+1}/v_t equals 1.

b. The value of money in economies A and B are, respectively,

\[ \frac{N(y - c_1^A)}{M} \quad \text{and} \quad \frac{N(y - c_1^B)}{M} \]

where c_1^A and c_1^B are first-period consumption in economies A and B, respectively. The assumption on preferences implies that c_1^A > c_1^B so that (y - c_1^A) < (y - c_1^B). This, in turn, implies that the value of money in economy A will be lower than the value of money in economy B. Intuitively, the demand for money will be larger in economy B than in economy A. This is because individuals in economy B want to hold relatively more money to finance their higher second-period consumption. Since all else is equal between the two economies (importantly, the supply of money and population), money will have a higher value in economy B than in economy A.