ECONOMICS 412 (SPRING 2008)
FINAL EXAMINATION (100 POINTS)

GAUTAM TRIPATHI

Please write clearly and show all your work. Points will be awarded only if the grader can understand what you have written. Answers without any explanation will not earn any points.

You have 2 hours. Good luck.

Problem 1 (20 Points)

Let \( Y_{it} = X_{it}' \theta_0 + C_i + \varepsilon_{it} \), where \( i = 1, \ldots, n \), \( t = 1, \ldots, T \), \( \theta_0 \in \mathbb{R}^p \), \( C_i \) is an unobserved random variable that may be correlated with \( X_{it} \), and \( \mathbb{E}[\varepsilon_{it}|X_{i1}, \ldots, X_{iT}, C_i] = 0 \) for each \( i, t \).

(i) Let \( \bar{Y} \), \( \bar{X} \), and \( \bar{\varepsilon} \) denote variables after the within transformation. For each \( i \), show that \( \mathbb{E}[\bar{\varepsilon}_{it}|X_{1}, \ldots, X_{T}, C] = 0 \) \( \implies \mathbb{E}[\bar{X}_{s}\bar{\varepsilon}_{t}] = 0 \) \( \forall s, t \) \( \implies \mathbb{E}[\bar{X}_{t}\bar{\varepsilon}_{t}] = 0 \) \( \implies T \sum_{t=1}^{T} \mathbb{E}[\bar{X}_{t}\bar{\varepsilon}_{t}] = 0 \).

(ii) Derive the closed form expression for the best estimator of \( \theta_0 \) under the assumption that \( \bar{X} \) is contemporaneously uncorrelated with \( \bar{\varepsilon} \).

(iii) What is its asymptotic distribution?

(iv) Suggest a GMM based test for the hypothesis \( \mathbb{E}[X_{it}C_i] = 0, t = 1, \ldots, T \).

Problem 2 (25 Points)

Consider the logit model \( \Pr(Y = 1|X) = \Lambda(X' \theta_0) \), where \( \Lambda(t) := e^t/(1 + e^t) \) for \( t \in \mathbb{R} \).

(i) Write down the joint density of \( Y \) and \( X \).

(ii) Does it matter for inference that the marginal distribution of \( X \) is unknown?

(iii) What is the score for a single observation? Simplify your answer as much as possible.

(iv) What is the asymptotic distribution of the MLE of \( \theta_0 \)?

(v) Describe how you would use the NTW statistic to test that the model is correctly specified.

Problem 3 (25 Points)

Suppose that \( Y \) is Bernoulli and \( \Pr(Y = 1|X) = \Phi(X' \theta_0) \).

(i) What is the interpretation of \( \beta_0 := \Pr(Y = 1|X = x_0) \), where \( x_0 \) is known?

(ii) Assume that \( \theta_0 \) is estimated by doing a probit of \( Y \) on \( X \) using the random sample \( (Y_1, X_1), \ldots, (Y_n, X_n) \). Use this information to propose a good estimator of \( \beta_0 \).

(iii) Let \( \hat{\beta} \) denote the estimator in (ii). What is the asymptotic distribution of \( \hat{\beta} \)?

(iv) What is the standard error of \( \hat{\beta} \)?

(v) Let \( p_0 \in (0, 1) \) be known. Suggest a size-\( \alpha \) test for \( \beta_0 \leq p_0 \) against \( \beta_0 > p_0 \).

Date: May 09, 2008.
Problem 4 (20 Points)

Let $Y_1 = 1(X'\theta_0 + Y_2\alpha_0 + \varepsilon > 0)$, where $\varepsilon|X \sim N(0, \sigma^2_\varepsilon)$ and $Y_2 \in \{0, 1\}$ may be endogenous. Furthermore, let $W$ denote the vector of instruments (included plus excluded) and $V$ be the reduced form error term such that $(\varepsilon, V)|W$ are jointly normal with mean zero and variance $\begin{pmatrix} \sigma^2_\varepsilon & \rho \sigma_\varepsilon \sigma_V \\ \rho \sigma_\varepsilon \sigma_V & \sigma^2_V \end{pmatrix}$.

(i) Write down the reduced form model for $Y_2$.
(ii) Which of the structural and reduced form parameters are identified?
(iii) Normalize the unidentified parameters to unity and write down the likelihood for a single observation. Use Result 1 in the appendix to minimize your calculations.
(iv) Suggest a test for $Y_2 \perp \varepsilon$.

Problem 5 (10 Points)

Let $Y \in \{0, 1\}$ and consider the binary choice model $\Pr(Y = 1|X) = F(X'\theta_0)$, where $F$ is a known cdf.

(i) Show that this model fully specifies all moments of $Y|X$.
(ii) Comment on the significance of this result.

Appendix

Result 1. Let $(\varepsilon, V)$ be jointly normal with mean zero and variance $\begin{pmatrix} \sigma^2_\varepsilon & \rho \sigma_\varepsilon \sigma_V \\ \rho \sigma_\varepsilon \sigma_V & \sigma^2_V \end{pmatrix}$. Then,

$$\Pr(a \leq \varepsilon \leq b|c \leq V \leq d) = \frac{1}{\sigma_V \left[ \Phi\left( \frac{d}{\sigma_V} \right) - \Phi\left( \frac{c}{\sigma_V} \right) \right]} \int_{v=c}^{d} \left[ \Phi\left( \frac{b - \frac{\rho \sigma_\varepsilon v}{\sigma_V}}{\sqrt{\sigma^2_\varepsilon (1 - \rho^2)}} \right) - \Phi\left( \frac{a - \frac{\rho \sigma_\varepsilon v}{\sigma_V}}{\sqrt{\sigma^2_\varepsilon (1 - \rho^2)}} \right) \right] \phi\left( \frac{v}{\sigma_V} \right) dv.$$