Cointegration: More on Testing Methods

Kremers, Ericsson, and Dolado (1992) discuss a potential problem with the ADF or DF test that holds for any single-equation unit root test. They argue that restrictions inherent in the method impose losses of information and therefore loss of power in the tests.

In summary, the Engle-Granger-type procedure:

- Is likely to have biased finite sample estimates of long-run relationships,
- Is likely to have lower power against alternative tests,
- Does not allow inferences using standard t-statistics.

Gregory and Hansen (1996) extend Engle-Granger to allow for a single break in the cointegration relationship.

An alternative to the Engle-Granger tests is the Cointegration Regression Durbin-Watson (CRDW) test of Sargan and Bhargava (1983). The test involves a simple regression of one variable on the other, and the standard Durbin-Watson test on the residuals. Recall the null for the DW test is that the residuals form a nonstationary random walk, whereas the alternative hypothesis is that the residuals form a stationary AR1 process. This is the most powerful test in the face of a trend. Unfortunately, the test only works when the residuals follow a first-order process (no higher-order autoregressivity in the residuals). This makes the CRDW unsuitable in all but very limited situations.

Kremers, Ericsson, and Dolado (1992) apply a test to the error-correction formulation of the model, with the null that the unit root exists. If the null cannot be rejected, then there is no cointegration. Under the null, the t-statistic has a non-normal distribution, and the McKinnon values must be used. Later researchers argued that the distribution is closer to normal than it is to the ADF distribution.

One problem with all single-equation models is that, depending on the number of variables in the model, there may be more than one cointegrating vector. Essentially, the tests assume that there is one unique vector. If there is more than one vector and only one is assumed, there are efficiency losses. We are trying to estimate a linear combination of the vectors when we use this method, giving a misleading error-correction model. Even when the assumption about a single vector is correct, estimating the single equation may be inefficient.

Highly seasonal data may exhibit seasonal unit roots. This implies that any cointegration with the series may occur at seasonal periods. In the face of seasonal unit roots, cointegration tests may give inconsistent estimates. In such cases the seasonals must be first removed before testing.

More generally, if the time series are highly periodic, then there may be stationary relationships between the variables that require a periodic cointegration model. Boswijk and Franses (1995) offer the model:

$$\Delta_4 y_t = \sum_{q=1}^{4} \alpha_q D_{qt} (y_{t-4} - \beta_q x_{t-4}) + \sum_{q=1}^{4} \mu_q D_{qt} + \sum_{q=1}^{4} \tau_q D_{qt} t$$

Where x_t is a vector of explanatory variables and D is a dummy variable corresponding to quarter q. The last two terms represent the seasonal intercepts and time trends which may or may not be included. Many researchers add lagged seasonal difference terms of x to better control for dynamics. The model requires nonlinear least squares.

The test of periodic cointegration involves the null that alpha and beta are zero using Wald tests.

We will discuss the Johansen test after we discuss VARs.