More on Cointegration

Examples of Cointegration from Economic Theory:

Consumption Theory

- **The permanent income hypothesis suggests that:**
  - **Transitory consumption is stationary**
  - **Consumption and permanent income are nonstationary (I(1)).**
  - **Therefore the linear combination of consumption and permanent income must be stationary.**

Unbiased Forward Rate Hypothesis

- **There must be a linear combination of (nonstationary) spot and forward exchange rates that is stationary.**

Key Points

1. **Cointegration refers to a linear combination of nonstationary variables. In theory, there are an infinite number of cointegrating vectors. Therefore, we usually normalize that cointegrating vector by fixing its length to 1.**

2. **Engle and Granger limit their original definition to variables that are integrated of the same order. If two integrated variables are of different order, then they cannot be cointegrated.**

   Consider two variables, one is I(1) and the other is I(2). They cannot be cointegrated. BUT if there is a linear combination of these two variables which is I(1), the linear combination CAN BE cointegrated with another I(1) variable.

The three variables are said to be cointegrated. Lee and Granger (1990) call this as multicointegration.

3. **It may be possible to have many independent cointegrating vectors. If multiple cointegrating vectors are found, behavioral relationships may be impossible to determine.**

4. **The literature in economics focuses on cases wherein the variables have single unit roots. That is, they are I(1). If two variables are I(2), then their first differences have single unit roots, and can be cointegrated. It is easier for economic interpretation to look at the variables this way than to attempt interpretation of the CI(2,1) cointegration.**

5. **Stock and Watson (1988) describe cointegrated variables as having the same stochastic trend. To restate this, the parameters of the cointegrating vector must be such that it purges any trend from the linear combination.**

   Note that the term "stochastic trend" is used to refer to the fact that each ε_t shock imparts a permanent random change to the conditional mean of the series. Thus the accumulation of these shocks amounts to a stochastic trend.

6. **The restrictions necessary to ensure that the variables are cointegrated guarantees that an error-correction model exists. The Granger Representation Theorem demonstrates that for any set of I(1) variables, error correction and cointegration are equivalent.**

7. **Johansen (1988) introduces another procedure for cointegration testing. It amounts to a multivariate extension of the Dickey-Fuller test.**

   In the univariate case, it is possible to view the stationarity of y_t as being dependent upon $(a_1 - 1)$:

$$y_t = a_1 y_{t-1} + \varepsilon_t$$

   or

$$\Delta y_t = (a_1 - 1) y_{t-1} + \varepsilon_t$$

   If $(a_1 - 1) = 0$, then y_t has a unit root. The Dickey-Fuller tables provide the statistics to test the null hypothesis that $(a_1 - 1) = 0$.

   To generalize to n variables, let x_t and ε_t and A_1 be an n x n matrix. Let

$$x_t = A_1 x_{t-1} + \varepsilon_t$$

   and

$$\Delta x_t = (A_1 - I) x_{t-1} + \varepsilon_t$$

   By analogy, if $(A_1 - I)$ consists of all zeros, so that it has rank zero, then all of the sequences in x_t are unit root processes. No linear combination of the sequences is stationary, so the variables are not cointegrated. This can be modified to include a drift term.

8. **Perron (1989) challenged the work of Nelson and Plosser (1983) by arguing that most economic variables are NOT best characterized by unit root processes. They are best represented as trend**

stationary processes with structural breaks. That is, they are piece-wise stationary. Such processes will appear to be unit-root processes when examined by DF and ADF tests.

9. **If series have unit roots, then Nelson and Plosser tell us that it is dangerous to "remove" deterministic trends from them. This results in spurious (false) fluctuations in the series.**

   A common way of decomposing a nonstationary series into a trend and a stationary component is the Hodrick-Prescott (1997) decomposition. Unfortunately, it is data dependent and also introduces spurious fluctuations. It attempts to smooth the change in mean over time.