

## BAND SPECTRUM REGRESSION

Engle, Robert F., "Band Spectrum Regression", *International Economic Review*, Vol. 15 No. 1 (February 1974).

First proposed by:

Hannan, E. J., "The Estimation of a Lagged Regression Relation," *Biometrika LIV*(December 1967), 409-418.

\_\_\_\_\_, "The Estimation of Relationship Involving Distributed Lags," *Econometrica* 33 (January 1965), 206-224.

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Problems, Reasons why more of this hasn't been done:

1. Requires large amounts of data.
2. Complex computations.
3. Involves "judgmental parameters" such as spectral windows and truncation points.
4. Uncertain payoff in terms of gains over time domain techniques.
5. Given that other researchers are relatively unfamiliar with such techniques, the results may be less persuasive than time domain techniques, and harder to place in leading mainstream journals.

Over the years, many of these concerns are less of a problem.

Engle argues:

1. Frequency domain techniques have the standard small sample properties.
2. Band Spectrum Regression does not require spectral windows.
3. Nowadays, these techniques are computationally easy to use.
4. These techniques lead to very natural and simple solutions to such difficult problems as errors in variables and seasonality.

Engle begins by defining the periodogram as an unbiased estimator of the (power) spectrum. In doing so, he cites:

Jenkins, G., and Donald G. Watts. *Spectral Analysis and Its Application*. San Francisco: Holden-Day, 1968.

He points out that the periodogram is asymptotically unbiased but inconsistent since the variance of each spectral estimator does not shrink as the sample becomes infinite.

This inconsistency leads to the use of spectral windows or periodogram averaging to obtain spectral estimators.

What are spectral windows?

- When we compute a periodogram, we do so from a finite data sample. Fortunately, this ensures that the power and energy are bounded and therefore that the integrals exist. But it also affects the estimates.
- If we take a data sample, divide it into half, and compute periodograms for each half, the two periodograms will have common broad features, but differ in fine detail. (Beveridge, 1921).
- The idea is that the power spectrum (spectral density) is reflected in these broad features, while the differences in finer detail are related to variations in individual samples.
- Some researchers have suggested simply dividing up the sample into as many subsamples as possible, computing a periodogram for each subsample, and then averaging them to form the spectral estimate. (Bartlett, 1948, in *Nature*)
- Another approach is compute the periodogram for the entire sample, and then to smooth the periodogram to form the spectral estimate.
- A common form is a weighted moving average (across a moving “window”). This is the spectral window.

For basic factors are taken into account when choosing a spectral window:

1. Resolution
2. Stability
3. Leakage

Resolution is the ability of the spectrum estimate to represent the fine structure in the frequency properties—narrow peaks, etc.

Stability is the extent to which estimates computed from different subsamples “agree”. Irrelevant fine structure is removed

Leakage refers to a low value being swamped by neighboring high values. More generally, it has to do with the spectrum at some frequency being made nonzero by a spectral value at another frequency.

Basically, Engle lets  $\tilde{x}$  be the Fourier transform of  $x$  times  $\sqrt{T}$  to transform:

$$y = x\beta + \varepsilon$$

into

$$\tilde{y} = \tilde{x}\beta + \tilde{\varepsilon}$$

noting that the latter equation is a regression equation with complex random variables.

Under the assumption that  $x$  and therefore  $\tilde{x}$  are independent of the error term, the Gauss-Markov theory implies that the BLUE of the second regression equation is:

$$\hat{\beta} = (\tilde{x}^* \tilde{x})^{-1} \tilde{x}^* \tilde{y}$$

None of this requires smoothed periodograms, and the estimator is consistent. It also turns out that any case of nonspherical disturbances in the time domain becomes heteroscedasticity in the frequency domain.

Engle points out that, in the time domain, it is very common to exclude some periods such as wars or strikes for various reasons. Why not do the same for frequencies in the frequency domain?

Perhaps a model can explain slow shifts but not fast ones?  
 Perhaps a model can explain season but not nonseasonal behavior?

Engle discusses using band spectrum regression to deal with seasonal adjustments, pointing out that you would want to zero the seasonal components AND their harmonics.

## PRACTICAL CONSIDERATIONS

Engle points out that most computer programs do not allow for complex data. The solution is to take the Fourier transform, select the frequency components, inverse transform, and then run the regression in the time domain.

In some cases, it may make sense to filter only the exogenous variable(s).

TABLE 1  
 ESTIMATES OF THE MARGINAL PROPENSITY TO CONSUME

	M.P.C.	St. Error	T-Stat	D. F.	SSR
All	.89541	.00293	305.4	101	1815.670
Transitory (high frequency)	.89397	.01056391	84.6	75.5	366.290
Permanent (low frequency)	.89545	.01095428	81.7	24.5	1449.230

$$F_{1,100} = .0065$$