MORE ON VARs

CONSIDER THE FOLLOWING SYSTEM:

$$y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}$$

- THIS IS A FIRST-ORDER VAR BECAUSE IT INCLUDES ONLY FIRST LAGS.
- IT IS ALSO REFERRED TO AS A STRUCTURAL VAR OR "THE PRIMITIVE SYSTEM".

THIS CAN BE WRITTEN IN MATRIX FORM AS:

$$\underbrace{\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} y_t \\ z_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}}_{\Gamma_0} + \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}}_{\Gamma_1} \underbrace{\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}}_{\varepsilon_t}$$

YIELDING:

$$B x_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

PREMULTIPLYING BY B^{-1} YIELDS:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

WHERE $A_0 = B^{-1}\Gamma_0$, $A_1 = B^{-1}\Gamma_1$, $e_t = B^{-1}\varepsilon_t$

THIS IS CALLED THE VAR MODEL IN STANDARD FORM.

NOTE THAT e_{1t} AND e_{2t} HAVE:

- ZERO MEANS
- CONSTANT VARIANCES
- INDIVIDUALLY SERIALLY UNCORRELATED (ALL AUTOCOVARIANCES ARE ZERO)
- BUT e_{1t} AND e_{2t} <u>ARE</u> <u>CORRELATED</u>.

<u>STABILITY</u>

FOR AN AR1 PROCESS TO BE STABLE,

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t,$$

$|a_1| < 0$, OR EQUIVALENTLY,

$$|a_1|^n \longrightarrow 0 \text{ AS } n \longrightarrow \infty.$$

SIMILARLY, FOR A VAR1 PROCESS

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

TO BE STABLE, $A_1^n \rightarrow 0$ AS $n \longrightarrow \infty$.

IF THE STABILITY CONDITION HOLDS, THEN THE SEQUENCES $\{y_t\}$ AND $\{z_t\}$ WILL BE JOINTLY COVARIANCE STATIONARY.

<u>PARSIMONY</u>

CLEARLY THE VAR WILL BE <u>OVERPARAMETERIZED</u> IN THAT MANY OF THE COEFFICIENT ESTIMATES WILL NOT BE SIGNIFICANTLY DIFFERENT FROM ZERO.

FOR THE GENERAL VAR MODEL, THE RHS CONTAINS ONLY PREDETERMINED VARIABLES, AND THE ERROR TERMS ARE SERIALLY UNCORRELATED WITH CONSTANT VARIANCE.

⇒ EACH EQUATION CAN BE ESTIMATED WITH OLS.

⇒ OLS ESTIMATES ARE CONSISTENT AND ASYMPTOTICALLY EFFICIENT.

☆ EVEN THOUGH ERROR TERMS ARE CORRELATED ACROSS EQUATIONS, SUR DOES NOT ADD TO THE EFFICIENCY SINCE ALL REGRESSIONS HAVE IDENTICAL RHS VARIABLES.

STATIONARITY OF VARIABLES

SIMS (1980) "MACROECONOMICS AND REALITY", ECONOMETRICA.

SIMS, STOCK, WATSON (1990). "INFERENCE IN LINEAR TIME SERIES MODELS WITH SOME UNIT ROOTS," ECONOMETRICA.

⇒ RECOMMEND AGAINST DIFFERENCING EVEN IF THE VARIABLES CONTAIN A UNIT ROOT.

THEY ARGUE:

- DIFFERENCING INVOLVES AN INFORMATION LOSS; ALSO DO NOT DETREND.
- PARAMETER ESTIMATES ARE IRRELEVENT.
- FORECASTING IS THE ISSUE

BY CONSTRUCTING THE VECTOR MOVING AVERAGE REPRESENTATION OF THE VARIABLES, WE GET:

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$$

THE COEFFICIENTS OF ϕ_i CAN BE USED TO GENERATE THE EFFECTS OF ε_{yt} AND ε_{zt} SHOCKS ON THE ENTIRE TIME PATHS OF $\{y_t\}$ AND $\{z_t\}$. THE INITIAL VALUES OF THE ϕ_i ARE THE IMPACT MULTIPLIERS. THE SUM OF THESE ACROSS ALL PERIODS ARE THE LONG-RUN MULTIPLIERS.

THUS THE VALUES OF THE ENTRIES IN ϕ_i ARE KNOWN AS IMPULSE RESPONSE FUNCTIONS.

PLOTTING $\phi_{jk}(i)$ AGAINST i IS A PRACTICAL WAY TO VISUALLY REPRESENT THE BEHAVIOR OF THE VARIABLES IN RESPONSE TO RANDOM SHOCKS.

THE CHOLESKI DECOMPOSITION PROVIDES THE NECESSARY RESTRICTIONS TO IDENTIFY THE IMPULSE RESPONSES. THIS DECOMPOSITION FORCES AN ORDERING ON THE VARIABLES, INVOLVING AN ASSUMPTION OF SOME VARIABLES BEING CAUSALLY PRIOR TO OTHERS.

☆ THE IMPORTANCE OF ORDERING DEPENDS ON THE MAGNITUDE OF THE CORRELATION BETWEEN THE ERRORS OF THE EQUATIONS IN STANDARD FORM.

VARIANCE DECOMPOSITION

THIS DECOMPOSES THE n-STEP-AHEAD FORECAST ERROR VARIANCE INTO THE PROPORTIONS DUE TO EACH SHOCK. THIS IS THE CHANGE IN EACH VARIABLE DUE TO ITS OWN SHOCKS VERSUS SHOCKS TO OTHER VARIABLES.

YOU WOULD EXPECT SHORT-RUN CHANGES TO BE DOMINATED BY SHOCKS TO THE VARIABLE IN QUESTION, WITH LONGER-RUN CHANGES BEING MORE AFFECTED BY SHOCKS TO OTHER VARIABLES.

NOTE THAT THE CHOLESKI DECOMPOSITION FORCES CHANGES IN THE ONE-PERIOD FORECAST ERROR. ALTERNATE ORDERINGS GENERATE GROSSLY DIFFERENT RESULTS AT SHORTER FORECAST HORIZONS.

IMPULSE ANALYSIS AND VARIANCE DECOMPOSITIONS ARE TOGETHER CALLED INNOVATION ACCOUNTING.

NEAR-VARS

IF YOU SET UP THE SYSTEM SO THAT SOME OF THE EQUATIONS HAVE DIFFERENT REGRESSORS, THE METHOD OF SEEMINGLY UNRELATED REGRESSIONS (SUR) PROVIDES EFFICIENT ESTIMATES OF THE COEFFICIENTS.