PARTIAL CORRELATION

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

Regress: $\hat{y} = \alpha_1 + \alpha_2 x_1$

$\hat{x}_1 = \gamma_1 + \gamma_2 x_2$

Form: $y^* = y - \hat{y}$

$x_1^* = x - \hat{x}_1$

THE CORRELATION BETWEEN y^* AND x_1^* IS THE PARTIAL CORRELATION BETWEEN y AND x_1.

$$\frac{\mathrm{cov}(y^*, x^*)}{\mathrm{var}(y^*)\mathrm{var}(x_1^*)}$$

IT IS THE EFFECT OF x_1 ON y NOT ACCOUNTED FOR BY x_2.

PARTIAL AUTOCORRELATION FUNCTION

CONSIDER THE AR(2) PROCESS $x_t = a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t$.

$$\text{LAG 1 ACF} = r_1 = \frac{\operatorname{cov}(x_t, x_{t-1})}{\operatorname{var}(x_t)}$$

$$= \frac{\operatorname{cov}(a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t, x_{t-1})}{\operatorname{var}(x_t)}$$

$$= a_1 \underbrace{\left[\frac{\operatorname{cov}(x_{t-1}, x_{t-1})}{\operatorname{var}(x_t)}\right]}_{\to 1} + a_2 \left[\frac{\operatorname{cov}(x_{t-2}, x_{t-1})}{\operatorname{var}(x_t)}\right] + \underbrace{\left[\frac{\operatorname{cov}(\varepsilon_t, x_{t-1})}{\operatorname{var}(x_t)}\right]}_{\to 0}$$

$$= a_1 + a_2 r_1$$

$$\text{LAG 2 ACF} = r_2 = a_1 r_1 + a_2$$

IN GENERAL, FOR AR(p), WE HAVE p EQUATIONS:

$$r_1 = a_1 + a_2 r_1 + a_3 r_2 + \cdots + a_p r_{p-1}$$
$$r_2 = a_1 r_1 + a_2 + a_3 r_1 + \cdots + a_p r_{p-2}$$
$$r_3 = a_1 r_2 + a_2 r_1 + a_3 + \cdots + a_p r_{p-3}$$
$$\vdots$$
$$r_p = a_1 r_{p-1} + a_2 r_{p-2} + a_3 r_{p-3} + \cdots + a_p$$

SOLVE FOR $a_1 = \hat{a}_1$ UNDER THE ASSUMPTION $p = 1$

LET $p=2$, SOLVE FOR $\hat{a}_1$ AND $\hat{a}_2 \rightarrow \hat{a}_2$

LET $p=3$, SOLVE FOR $\hat{a}_1, \hat{a}_2,$ AND $\hat{a}_3 \rightarrow \hat{a}_3$

ETC.

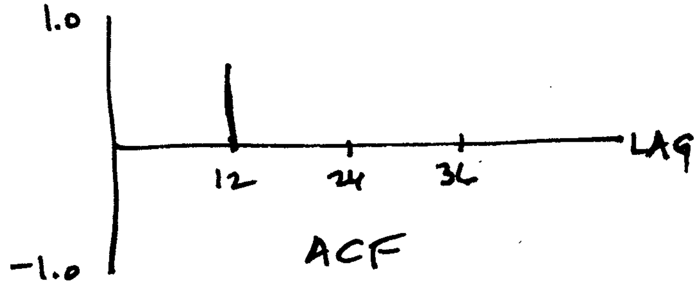
$\hat{a}_1, \hat{a}_2, \hat{a}_3, \ldots$ ARE THE <u>PARTIAL AUTOCORRELATIONS</u>.

THE SAMPLE ACF IS ESTIMATED BY FITTING AR PROCESSES OF SUCCESSIVELY HIGHER ORDER. COLLECT THE $\hat{a}_i$ WHEN THE iTH $AR(i)$ IS FITTED. VALUES OUTSIDE $\pm 2/\sqrt{N}$ ARE SIGNIFICANTLY DIFFERENT FROM ZERO AT THE 5% LEVEL.

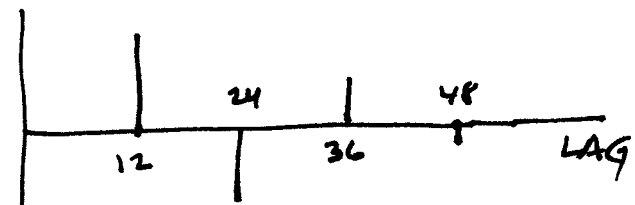
WHEN FITTING AN $AR(p)$ MODEL, THE LAST COEFFICIENT MEASURES THE EXCESS CORRELATION AT LAG p, WHICH IS NOT ACCOUNTED FOR BY AN $AR(p-1)$ MODEL. THIS IS THE pTH PARTIAL AUTOCORRELATION FUNCTION.

SAMPLE ACF AND PACFS

EXAMPLE 1. SMA (1) ORDER 12; $x_t = 0.7\varepsilon_{t-12} + \varepsilon_t$

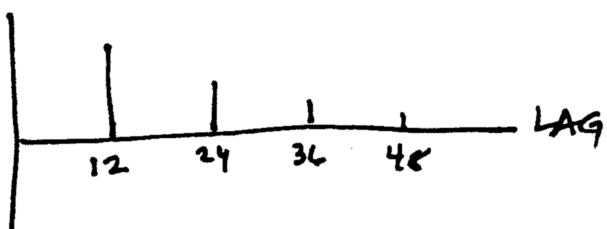


ACF

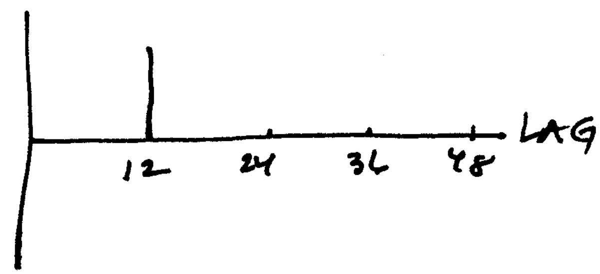


PACF

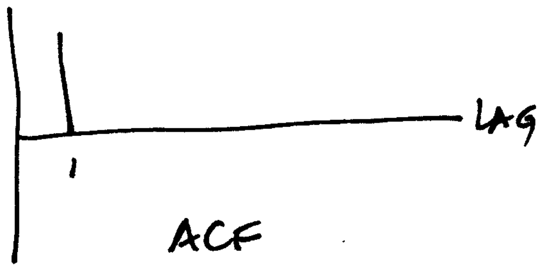
EXAMPLE 2. SAR (1) ORDER 12; $x_t = 0.7x_{t-12} + \varepsilon_t$



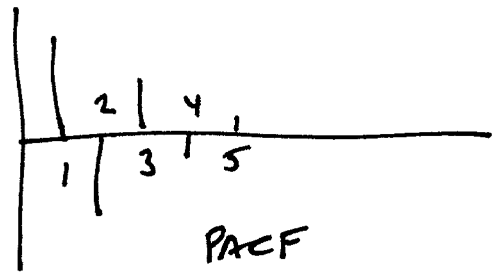
ACF



EXAMPLE 3. MA(1); $x_t = 0.7\varepsilon_{t-1} + \varepsilon_t$

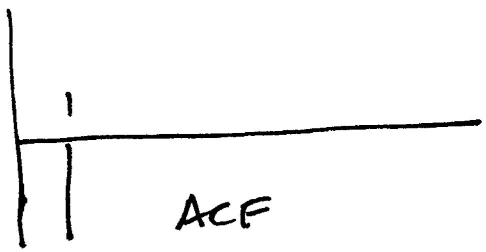


ACF



PACF

EXAMPLE 4. MA(1); $x_t = -0.7\varepsilon_{t-1} + \varepsilon_t$



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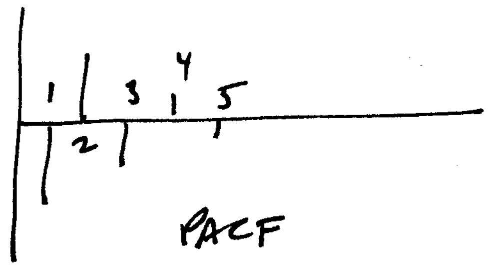
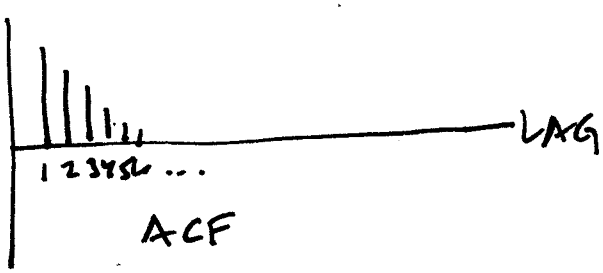
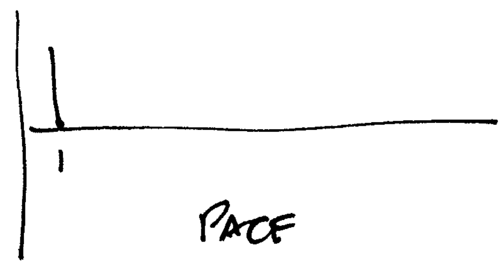


PACF

EXAMPLE 5. AR(1); $x_t = 0.7x_{t-1} + \varepsilon_t$



ACF



PACF

SUMMARY	ACF	PACF
MA (q)	SPIKES AT LAGS 1 THROUGH q, ZEROES ELSEWHERE	SPIKES DECREASE EXPONENTIALLY. BEGIN AT LAG 1. FOR $q > 1$, DAMPED SINE WAVES ON THE PATTERN.
AR (p)	SPIKES DECREASING EXPONENTIALLY BEGIN AT LAG 1. FOR $p > 1$, SINE WAVES ON PATTERN.	SPIKES AT LAGS $1 \rightarrow p$. ZEROS ELSEWHERE,
ARMA(p,q)	IRREGULAR SPIKES AT LAGS $1 \rightarrow q$. REMAINING PATTERN AS IN AR.	IRREGULAR PATTERN OF SPIKES AT LAGS $1 \rightarrow p$; REMAINING PATTERN AS IN MA.

FITTING AR PROCESSES

FIT AR PROCESSES OF PROGRESSIVELY HIGHER ORDER,
CALCULATE RESIDUAL SUM OF SQUARES FOR EACH.
PLOT AGAINST ORDER.
WHERE CURVE "FLATTENS OUT" — QUIT.
(THERE IS LITTLE IMPROVEMENT FOR NCREASED ORDER.)

FITTING MA PROCESSES

FORM MA PROCESS SIMILARLY, FORM A GRID SEARCH, LOOKING FOR LOWEST RSS.