Comparative Static Analysis of the Keynesian Model

Macroeconomics I

ECON 309 -- Cunningham

Simple IS-LM Analysis

$$S(Y) - I(r) - G = 0$$

$$L(Y, r) - \frac{M}{P} = 0$$

Two equations, two endogenous variables (*Y* and *r*), and one exogenous variable *G*. Real money supply (*M/* P) is taken as constant since nominal money (*M*) and (*P*) are exogenous as well.

Take total differentials:

$$S_Y dY - I_r dr = dG$$

$$L_Y dY + L_r dr = 0$$

Write in matrix form:

$$\begin{bmatrix} S_Y & -I_r \\ L_Y & L_r \end{bmatrix} \begin{bmatrix} dY \\ dr \end{bmatrix} = \begin{bmatrix} dG \\ 0 \end{bmatrix}$$

Simple IS-LM, Continued

Applying Cramer's rule for solution:

$$\frac{dY}{dG} = \frac{\begin{vmatrix} 1 & -I_r \\ 0 & L_r \end{vmatrix}}{\begin{vmatrix} S_Y & -I_r \\ L_Y & L_r \end{vmatrix}} = \frac{L_r}{S_Y L_r + I_r L_Y} > 0$$

$$\frac{dr}{dG} = \frac{\begin{vmatrix} S_Y & 1 \\ L_Y & 0 \end{vmatrix}}{\begin{vmatrix} S_Y & -I_r \\ L_Y & L_r \end{vmatrix}} = \frac{-L_Y}{S_Y L_r + I_r L_Y} > 0$$

So, in a Keynesian economy, under the conditions given, cet. par. (i.e., prices), an increase in government spending increases GDP and interest rates.

Because, by assumption, the following hold:

$L_r < 0,\ L_Y > 0$

$S_Y > 0,\ I_r < 0$

Extension

What if prices are flexible? To examine this, we must include the labor market and real wage computation.

$$N^d\left(\frac{\overline{w}}{P}\right) - N = 0$$

$$Y - F(N) = 0$$

$$S(Y) - I(r) - G = 0$$

$$L(Y, r) - \frac{M}{P} - 0$$

Define the following variable as a convenience:

$$X = \frac{\partial N^d}{\partial\left(\overline{w}/P\right)}\left(-\frac{\overline{w}}{P^2}\right) > 0$$

Extension (Continued)

$$\frac{dY}{dG} = \frac{\begin{vmatrix} 0 & -1 & 0 & X \\ 0 & -F_N & 0 & 0 \\ 1 & 0 & -I_r & 0 \\ 0 & 0 & L_r & \frac{M}{P^2} \end{vmatrix}}{\begin{vmatrix} 0 & -1 & 0 & X \\ 1 & -F_N & 0 & 0 \\ S_Y & 0 & -I_r & 0 \\ L_Y & 0 & L_r & \frac{M}{P^2} \end{vmatrix}} = \frac{-L_r F_N X}{|Jac|} > 0$$

Similarly:

$$\frac{dN}{dG} = -\frac{L_r X}{|Jac|} > 0$$

$$\frac{dr}{dG} > 0$$

$$\frac{dP}{dG} > 0$$

$$\frac{d\left(\frac{\overline{W}}{P}\right)}{dG} < 0$$

(Note that the denominator turns out to be positive.)