Comparative Statics:
Neoclassical Model

Graduate Macroeconomics I

ECON 309 – Cunningham

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Method of Comparative Statics

Given, an economic system of the form:

$$f_i(x_1, x_2, \ldots, x_n) = 0,\ i = 1, 2, 3, \ldots, n$$

Such a system can be interpreted as the set of certain equilibrium relations or the set of certain optimization conditions. If equilibrium, then f_i and x_i are the excess demands for and prices of the ith commodity.

To consider "shifts" of the system due to changes in exogenous variables, rewrite the system making the exogenous variables explicit:

$$f_i(x_1, x_2, \ldots, x_n; a, b, \ldots).$$

Comparative Statics, continued

Assume all second partials exist and are continuous. Then $a, b, \ldots$ are the “shift parameters” or exogenous variables. If the f_i are well-behaved, then we can write

$$x_i = x_i(a, b, \ldots),\ i = 1, 2, \ldots, n.$$

Comparative statics is concerned with the effect of a change in one or more of the shift parameters on the equilibrium values of the x_i.

In effect the equations are linearized around the current equilibrium values of the exogenous variables by taking total differentials. The implied assumption is that the changes will be small and in some neighborhood of the original equilibrium.

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Example 1

Question: According to the neoclassical model, what happens to output, wages, and employment when the capital stock is increased?

Solution: construct as simple a formal model as possible that can examine this question. (Parsimony)

$$Y = F(N, K)$$

$$\frac{w}{P} = F_N$$

$$N^S = N^S\left(\frac{w}{P}\right)$$

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Example 1, Continued

Make the usual assumptions:

$$F_N = \frac{dF}{dN} > 0,\ F_{NN} = \frac{d^2F}{dN^2} < 0$$

$$F_K = \frac{dF}{dK} > 0,\ F_{KK} = \frac{d^2F}{dK^2} < 0$$

$$F_{NK} = F_{KN} = 0$$

Take total differentials:

$$dY = F_N dN + F_K dK$$

$$d\left(\tfrac{w}{P}\right) = F_{NN} dN + F_{NK} dK$$

$$dN = N_{\frac{w}{P}} d\left(\tfrac{w}{P}\right)$$

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Example 1, Continued

Reorganize:

$$\begin{aligned} dY &\quad +0 &\quad -F_N dN &\quad = \quad F_K dK \\ 0 &\quad +d\left(\tfrac{w}{P}\right) &\quad -F_{NN} dN &\quad = \quad F_{NK} dK \\ 0 &\quad +N_{\frac{w}{P}} d\left(\tfrac{w}{P}\right) &\quad +dN &\quad = \quad 0 \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 1 & 0 & -F_N \\ 0 & 1 & -F_{NN} \\ 0 & -N_{\frac{w}{P}} & 1 \end{bmatrix} \begin{bmatrix} dY \\ d\left(\tfrac{w}{P}\right) \\ dN \end{bmatrix} = \begin{bmatrix} F_K dK \\ F_{NK} dK \\ 0 \end{bmatrix}$$

Jacobian (J) Endogenous Variables Exogenous Variables

Example 1, Continued

Find the determinant of the Jacobian.

$$\det J = |J| = 1 - \overset{(-)}{F_{NN}} \overset{(+)}{N_{\frac{w}{P}}} > 0$$

Apply Cramer's Rule, replacing column 1 with the vector of exogenous variables (RHS).

$$dY = \frac{\begin{vmatrix} F_K dK & 0 & -F_N \\ F_{NK} dK & 1 & -F_{NN} \\ 0 & -N_{\frac{w}{P}} & 1 \end{vmatrix}}{1 - F_{NN} N_{\frac{w}{P}}}$$

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Example 1, Continued

Expand by minor determinants, third row:

$$dY = \frac{N_{\frac{w}{P}} \begin{vmatrix} F_K dK & -F_N \\ F_{NK} dK & -F_{NN} \end{vmatrix} + \begin{vmatrix} F_K dK & 0 \\ F_{NK} dK & 1 \end{vmatrix}}{1 - F_{NN} N_{\frac{w}{P}}}$$

$$dY = \frac{N_{\frac{w}{P}} \left[F_N F_{NK} dK - F_K F_{NN} dK \right] + F_K dK}{1 - F_{NN} N_{\frac{w}{P}}}$$

Example 1, Continued

$$\frac{dY}{dK} = \frac{N_{\frac{w}{P}} F_N \overset{0}{F_{NK}} - \overset{(+)}{N_{\frac{w}{P}}} \, \overset{(+)}{F_K} \, \overset{(-)}{F_{NN}} + \overset{(+)}{F_K}}{1 - \underset{(-)}{F_{NN}} \, \underset{(+)}{N_{\frac{w}{P}}}} \quad > 0$$

Ceteris paribus, an increase in the capital stock results in an increase in output, under normal assumptions.

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Example 1, Continued

What happens to real wages? Replace column 2 to find out.

$$d\left(\tfrac{w}{P}\right) = \frac{\begin{vmatrix} 1 & F_K dK & -F_N \\ 0 & F_{NK} dK & -F_{NN} \\ 0 & 0 & 1 \end{vmatrix}}{(+)} = \frac{F_{NK} dK}{(+)}$$

$$\frac{d\left(\tfrac{w}{P}\right)}{dK} = \frac{F_{NK}}{(+)} = 0 \qquad \text{(Positive only if } F_{NK} > 0\text{.)}$$

Cet. par., an increase in the capital stock has no effect on real wages unless some of the productivity of the additional capital accrues to labor. (The increase in *K* would have to cause increase labor marg. productivity.)

Example 1, Continued

$$dN = \frac{\begin{vmatrix} 1 & 0 & F_K dK \\ 0 & 1 & F_{NK} dK \\ 0 & -N_{\frac{w}{P}} & 0 \end{vmatrix}}{(+)} = \frac{N_{\frac{w}{p}} F_{NK} dK}{(+)}$$

$$\frac{dN}{dK} = \frac{N_{\frac{w}{p}} F_{NK}}{(+)} = 0 \quad \text{(Positive only if } F_{NK} > 0.\text{)}$$

Cet. par., changes in the capital stock have no impact on employment unless it improves the marginal productivity of labor.

Example 2

Question: in the neoclassical model, for a given level of output, what is the effect of a change in the money supply?

$$M = kPY$$

$$\frac{M}{P} = kY$$

$$d\left(\frac{M}{P}\right) = \frac{PdM - MdP}{P^2} = kdY$$

$$PdM - MdP = P^2 kdY$$

$$P - \frac{MdP}{dM} = P^2 k \frac{dY}{dM},\ but\ \frac{dY}{dM} = 0$$ by neutrality.

Example 2, Continued

$$P - \frac{MdP}{dM} = 0$$

$$M\frac{dP}{dM} = P$$

$$\frac{dM}{M} = \frac{dP}{P}$$

Cet. par., the inflation rate is equal to the growth rate of the money supply.