Solution to Exercise 3

1) At steady-state,

\[ C^* = (1 - \delta) f(k^*_0), \quad \text{let } \delta = 1 \]

where \( C^* = \) steady-state level of consumption,
\( k^*_0 = \) steady-state capital per worker ratio.

\[ \text{Row } C^* = f(k^*_0) - \alpha f(k^*_0) \]

but at steady-state \( \alpha f(k^*_0) = (n + d) k^*_0 \)

so we can rewrite

\[ C^* = f(k^*_0) - (n + d) k^*_0 \]

For the planner who wishes to maximize consumption per capita at steady state, the objective function is

\[ \max \quad C^* = f(k^*_0) - (n + d) k^*_0 \]
\[ k^*_0 \]

The 1st-order condition is

\[ f'(k^*_0) - (n + d) = 0 \quad \text{or} \quad f'(k^*_0) = (n + d), \quad \text{which is also the Golden-rule steady state.} \]
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2. The only impact effect of this disturbance is to lower the capital stock. Therefore, the production possibility frontier shifts down and the marginal product of labor falls (PPF is flatter).

(a) The reduction in the capital stock is depicted in the figure below. The economy starts at point A on PPF1. The reduction in the capital stock shifts the production possibilities frontier to PPF2. Because PPF2 is flatter, there is a substitution effect that moves the consumer to point D. The consumer consumes less of the consumption good and consumes more leisure, increasing labor supply. Because the production possibilities frontier shifts down, there is also an income effect. The income effect implies less consumption and less leisure (more work). On net, consumption must fall, but leisure could decrease, remain the same, or increase, depending on the relative strengths of the income and substitution effect. The real wage must also fall. To see this, we must remember that, in equilibrium, the real wage must equal the marginal rate of substitution. The substitution effect implies a lower marginal rate of substitution. The income effect is a parallel shift in the production possibilities frontier. As the income effect increases the amount of employment, marginal product of labor must fall from point D to point B. This reinforces the reduction in the marginal rate of substitution from point A to point D.
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(26) From the 1st-order condition

\[ u_x(\xi t) - u_1 \xi \bar{F}_2(x, \xi t) = 0 \]
\[ \bar{F}(x, \xi t) - \xi - c = 0 \]

Total differential:

\[
\begin{bmatrix}
-1 & -\xi \bar{F}_2 \\
\xi \bar{F}_2 u_{11} & \xi \bar{F}_2 u_{22} + \xi \bar{F}_2 u_1 u_{22}
\end{bmatrix}
\begin{bmatrix}
dc \\
d\xi
\end{bmatrix}
= 
\begin{bmatrix}
-F d\xi + d \xi \xi \bar{F}_2 d\xi - \xi \bar{F}_2 d\xi \\
u_1 \bar{F}_2 d\xi + u_1 \bar{F}_2 \xi d\xi
\end{bmatrix}
\]

\[
\frac{dc}{d\xi} = -\xi \bar{F}_2 (u_{22} - \xi \bar{F}_2 u_{12} + u_1 \xi \bar{F}_{22})
\]

\[
\nabla = -\xi ^2 \bar{F}_2^2 u_{11} + \xi \bar{F}_2 (u_{12} - \xi \bar{F}_2 u_{11}) > 0
\]

\[
\frac{de}{d\xi} = \frac{-u_1 \bar{F}_2}{\xi} + \frac{\xi \bar{F}_1 (u_{12} - \xi \bar{F}_2 u_{11})}{\xi}
\]

Substitution effect:

\[
\frac{de}{d\xi} \bigg|_{sub.} = \frac{-u_1 \bar{F}_2}{\xi} < 0
\]

Income effect:

\[
\frac{de}{d\xi} \bigg|_{income} = \frac{\xi \bar{F}_1 (u_{12} - \xi \bar{F}_2 u_{11})}{\xi} > 0
\]
\[ W = z F_2 (x, h - c). \]

\[ dw = z F_{21} \, dx - z F_{22} \, dc + \Gamma \, dz \]

\[ \frac{dW}{dk} = z F_{21} - z F_{22} \frac{dc}{dk} \]

\[ \Gamma \begin{array}{c} \text{\textless} 0 \quad \text{See note next page} \\
\end{array} \]

where \( \nabla = -z x^2 F_{2} u_{11} + z x F_{2} u_{12} - u_{22} - z F_{22} u_{1} > 0 \)

\[ Y = C + G \]

\[ \frac{dY}{dk} = \frac{dc}{dk} + \frac{dG}{dk} \quad \frac{dG}{dk} = 0 \]

\[ = \frac{dc}{dk} = \frac{z F_{2} (u_{22} - z F_{2} u_{12} + u_{1} z F_{22}) + z F_{2} u_{1} z F_{21}}{<} \]

\[ > 0 \]
\[ \frac{dx}{dt} < 0 \]

The two terms:

- First term:

\[ \frac{1}{2} \sum \frac{2}{2} - \frac{1}{2} \sum \frac{2}{2} \]

- Second term:

\[ \frac{1}{2} \sum \frac{2}{2} - \frac{1}{2} \sum \frac{2}{2} \]

Now, take the 1st term:

\[ \sum \frac{2}{2} = \left( \sum \frac{2}{2} \right) \]

\[ \Delta \]

They cancel out!

\[ \sum \frac{2}{2} > 0 \]
(a) The increase in government spending in this example has two separate effects on the production possibilities frontier. First, the increase in government spending from $G_1$ to $G_2$ implies a parallel downward shift in the production possibilities frontier. Second, the productive nature of government spending is equivalent to an increase in total factor productivity that shifts the production possibilities frontier upward and increases its slope. The figure below draws the original production possibilities frontier as $PPF_1$ and the new production possibilities frontier as $PPF_2$. If the production-enhancing aspects of the increase in government spending are large enough, representative consumer utility could rise, as in this figure. [Note: This is not the only outcome. For example, $PPF_2$ could end up below $PPF_1$, so that $I_2$ is below $I_1$.]

(b) There are three effects at work in this example. First, there is a negative income effect from the increase in taxes needed to pay for the increased government spending. This effect tends to lower both consumption and leisure. Second, there is a substitution effect due to the productive effect of the increase in $G$, which is drawn as the movement from point A to point D. This effect tends to increase both consumption and leisure. Third, there is a positive income effect from the increase in $G$ on productivity. This effect tends to increase both consumption and leisure. In the figure above, the movement from point D to point B is the net effect of the two income effects. In general, consumption may rise or fall, and leisure may rise or fall. The overall effect on output is the same as in any increase in total factor productivity. Output surely rises.

(c) Note: there is no part (c) to this problem!