Notes to Chapter 6

Here are a couple of numerical examples that incorporate human capital (education) into the basic Solow model.

1. Suppose an economy uses a Cobb-Douglas production function with \( \alpha = 1/3 \), and has saving rate of 10%, a population growth rate of 2%, and a 5% physical capital depreciation rate. Suppose the average level of schooling for this economy is 7 years, find:

   a. the steady-state level of income per capita
   b. the contribution of human capital to steady-state income per capita.

Solution:

Use

\[
y'' = h \left\{ A^{\frac{1}{1-\alpha}} \left( \frac{y}{n+\delta} \right)^{\alpha (1-\alpha)} \right\}, \text{ with } A = 1. \text{ The first step is to find } h. \text{ For raw labor, or 0 year of schooling, } h = 1. \text{ For 7 years of schooling, we use a rate of return to education of 13.4\% per year for 1–4 years, and 10.1\% per year for 5–8 years. Thus, } h = (1.134)^4 \times (1.101)^3 = 2.21. \text{ Thus, the steady-state income per capita for an economy with an average of 7 years of education is}
\]

\[
y'' = 2.21 \left[ \frac{0.1}{0.02 + 0.05} \right]^{1/2} = 2.64.
\]

Without human capital, \( y'' = 1.20 \). Human capital increases income per capita by \( (2.64 - 1.20) = 1.44 \), or \( (2.64/1.20) = 2.21 \), income per capita with human capital is 221% of the income per capita without human capital.

2. Suppose an economy has 100 workers, 60 of the workers have 0 year of schooling while 40 of the workers have 7 years of schooling. (Note: this is a slight variation of the numerical example given in the textbook.)

There are two ways to ask this question:

a. Calculate the percentage of the wage bill that is going to human capital, and the percentage of the wage bill that is earned by raw labor (use 1 as the return to raw labor), and suppose \( \alpha = 1/3 \), calculate human capital’s share of the national income, and raw labor’s share of the national income.

b. Calculate the percentage of the wage bill that is going to workers with 0 year of schooling (use 1 as the return to raw labor), and the percentage of the wage bill that is earned by workers with human capital. Suppose \( \alpha = 1/3 \), calculate the share of the national income earned by workers with human capital, and the share of the national income earned by workers with no schooling.

Solution:

(a) First, there are 100 workers, and therefore the wage bill for raw workers is 100. Workers with 7 years of schooling earn 2.21 = (1.134)^4 \times (1.101)^3, but 1 is for raw labor, and therefore return to education is 1.21 per worker. The total wage bill for human capital is 40 x 1.21 = 48.4. Together, the total wage bill is (100 + 48.4) = 148.4, (100/148.4) = 0.67 (67\%) is the percentage of the wage bill going to raw labor, and (48.4/148.4) = 0.33 (33\%) is the percentage of the wage bill earned by human capital. Raw labor’s share of national income is (0.67 x 0.67) = 0.45 (45\%), and human capital’s share of national income is (0.67 x 0.33) = 0.22 (22\%).

(b) Workers with 0 year of schooling earn 1 while workers with 7 years of schooling earn 2.21 (from question 1 above). The total wage bill is (60 x 1) + (40 x 2.21) = 60 + 88.4 = 148.4. The percentage of the wage bill earned by workers with no schooling is (60/148.4) = 0.40, or 40\%, and therefore the percentage of the wage bill earned by workers with 7 years of schooling is (88.4/148.4) = 0.60 (60\%). The share of national income earned by workers with no education is (0.67 x 0.40) = 0.27, and the share of national income earned by workers with 7 years of schooling is (0.67 x 0.60) = 0.40.