Model mis-specification and Johansen’s co-integration analysis: an application to the US money demand

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Received 13 January 2000; accepted 18 April 2001

Abstract

This paper examines the consequences of mis-specifying the deterministic components of a co-integration model estimated with the Johansen’s multivariate maximum likelihood approach. Using the US long-run money demand model as our example, we show that when a linear deterministic time trend is excluded from the co-integration model, we obtain very robust results, suggesting the co-integration of the real M1 and the real M2 with their determinants. When a linear deterministic time trend is included, however, we find generally unfavorable results. Next, using a procedure discussed in Johansen (1992) that jointly determines the co-integration rank and the deterministic components of a model, we find that the preferred models for the real M1 and the real M2 are generally not co-integrated with their determinants. Our study suggests that great care should be exercised in the initial stages of model specification. The inclusion or exclusion of the linear deterministic time trend should be justified formally. Otherwise, the results may be misleading. © 2002 Elsevier Science Inc. All rights reserved.

JEL classification: E41; C51
Keywords: Co-integration; US money demand

1. Introduction

Johansen’s (1988) multivariate maximum likelihood approach to co-integration is arguably the most popular approach in estimating long-run economic relationships.
Studies comparing Johansen’s approach to other approaches in co-integration analysis have generally concluded favorably for the Johansen’s approach, although it should be noted that this is not necessarily true in each and every instance. For example, in a Monte Carlo study comparing Johansen’s approach to four other approaches of estimating a long-run equilibrium relationship, Gonzalo (1994) concludes that Johansen’s approach performs better than the other four approaches even when the errors are not normally distributed, or the dynamics of the vector error-correction model (VECM) are unknown, and additional lags are included in the VECM. In another Monte Carlo study, Hargreaves (1994) compares Johansen’s approach to five other methods of estimating long-run relationships and concludes that the Johansen’s method is the best if the sample size is fairly large (about 100 observations or more), the model is well specified, and the residuals are not highly autocorrelated. On the other hand, in a study on the long-run money demand relationship in the US using Johansen’s and several other approaches, Stock and Watson (1993) find that Johansen’s approach is sensitive to the lag length used in the VECM and to the sample-ending point.

Because of its popularity, a close scrutiny of Johansen’s approach is warranted. We report in this study the dramatically different results obtained from using Johansen’s approach when the models under investigation differ only in the deterministic components. In particular, we show that when a linear deterministic time trend is not included in the US money demand equation, we find strong evidence in favor of co-integration. When a linear deterministic time trend is included, however, we find dramatically weaker evidence in favor of co-integration. Thus, our paper is different from the studies cited above in that, rather than focusing on the properties of the estimators from the various approaches, we use only Johansen’s approach and focus on how the co-integration results of the long-run US money demand model may differ depending on the deterministic components used in the model.

The main objective of co-integration analysis is to determine the co-integration rank of the model. Considerable attention is focused on modeling the economic relationship of the variables, for example, what variables to include in the model. Johansen (1995), and Hansen and Juselius (1995), however, have emphasized that the choice of the deterministic components of a model has important implications for the asymptotic distribution of the test statistics. They provide five tables of critical values corresponding to the five different ways that the deterministic components could be included in a Johansen co-integration model. Quite clearly, empirical results

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1 The idea that empirical results may be dependent on how the deterministic components are included in the model is not entirely new. See Stock and Watson (1989), and Krol and Ohanian (1990) for an analysis in the “Granger causality” framework, and the references contained in these two studies for other applications. To our knowledge, however, this issue has not been examined empirically in a co-integration model framework, although the issue is well known at the theoretical level.

2 Stock (1994, Section 3.2.5) also discussed the consequences of misspecifying the deterministic components in co-integration models. Once again, this is not new in the empirical literature nor unique to co-integration models. For example, in unit-root testing, Fuller (1976), and Dickey and Fuller (1981) provide different tables of critical values depending on the deterministic components included in the model.
can potentially differ depending on the deterministic components included in the model. It is therefore rather surprising that, in actual application, the modeling of the deterministic components of co-integration models has received relatively little attention. ³ For example, in estimating co-integration models of money demand using the Johansen’s approach, a constant term is frequently included without much discussion (for example the papers by Hoffman and Rasche, 1991; Hoffman et al., 1995). In the studies by Choudhry (1996) and Miyao (1996), they report including a linear deterministic time trend in their co-integration models of money demand. Swanson (1998) reports detrending his data both with linear and quadratic trends. Finally, Carlson et al. (2000) report including a linear deterministic trend or a quadratic trend in their money demand specifications. None of these researchers, however, provide formal justification for their treatment of the deterministic components. Moreover, even if the researchers report including a constant and a linear or quadratic deterministic time trend in their co-integration models, it is frequently difficult to infer how these deterministic components are actually included in their models. The reason is because the discussion of ‘constants’ or ‘trends’ in Johansen’s co-integration model is only meaningful if they are related to the co-integration space or excluded from it, as we will make clear in the next section. However, this is rarely done in actual applications. In sum, we believe that the treatment of the deterministic components in a co-integration model has not been addressed adequately in the empirical literature. The consequences of mis-specifying the deterministic components in actual applications are largely unknown.

Although we believe that the results in our paper have general applicability to all research on long-run economic relationships using the Johansen’s approach, our use of the US money demand in this study is motivated by several factors. First, the importance of a stable money demand function as a prerequisite for the use of monetary aggregates in the conduct of monetary policy is well established in the monetary economics literature. For example, citing partly the instability of the relationship between M1 and economic activity, the Federal Reserve System (FED) abandoned M1 as a monetary target in early 1987, and switched to M2. In the early 1990s, however, again citing the breakdown in the relationship between M2 and economic activity, the FED also abandoned M2 as a monetary target. ⁴ Second, since Goldfeld (1976) found that his estimated short-run M1 money demand equation was systematically over-predicting the actual M1, the question of whether the long-run money demand equation remains a stable one has become a focus of research. Much of the latter body of literature have used the techniques of co-integration to assess whether a long-run equilibrium relationship exists between money demand and its determinants, which typically include an interest rate variable and a scale variable such as real income. The results thus far, however, have been mixed and do not appear to be robust. For example, Hoffman and Rasche (1991); King et al. (1991); Stock and Watson (1993); Hoffman et al. (1995), and Carlson et al. (2000), provide

³ The study by King et al. (1991) is an exception, however.
⁴ These are announcement dates. The actual dates of implementation are most certainly earlier.
evidence to support the co-integration of the money demand equation. Miller (1991) and Hafer and Jansen (1991), find that their co-integration results are sensitive to the monetary aggregates used. Choudhry (1996) finds that his results are not robust with respect to different specifications of the money demand equations. Friedman and Kuttner (1992) and Miyao (1996) find that their results are sensitive to the sample periods used. Finally, Swanson (1998) reports mixed results depending on the monetary aggregates used, the lag structure, and whether or not the data are linearly or quadratically detrended. Interestingly, Swanson (1998) appears to be the first one to note, that in empirical application, there is a dependency of the co-integration results on the assumed deterministic trend component in the model. He went on to suggest that “care needs to be taken at the initial model specification stage of our analysis” (pp. 463–464). Curiously, Swanson himself makes no attempt to determine which trend component is appropriate for his models.

Our paper examines the impacts on the empirical results of the different treatments of the deterministic components in co-integration models of the US money demand. Our main message is that dramatically different results could be obtained from seemingly similar models differing only in the deterministic components. Thus, we suggest that there is a need for a more careful modeling of the deterministic components of long-run economic models than had been the case in the past.

This paper has five remaining sections. We discuss briefly the five different ways that deterministic components can be incorporated in co-integration models in the next section. In Section 3, we discuss the US money demand equation and our data set. In Section 4, we present our preliminary empirical results where we examine the time series properties of the variables used in this study. Section 5 contains our co-integration results, and Section 6 contains our summary and conclusions.

2. Brief review of the deterministic components in a co-integration model

Since Johansen’s co-integration model is now well known, we will focus our attention on modeling the deterministic components of the model. We start by writing the \( p \)-dimensional vector autoregressive model in its VECM form:

\[
\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \alpha \beta' z_{t-1} + \mu + \Psi D_t + \varepsilon_t,
\]

where \( \Delta \) is the first difference operator, \( z_t \) is a \( px1 \) vector of stochastic variables, \( \alpha \) and \( \beta \) are \( pxr \) matrices of full rank, \( \mu \) is a vector of constants, \( D_t \) is a vector of deterministic variables, which may include linear time trends, seasonal dummies, etc., and \( \varepsilon_t \) is a vector of normally, independently, and identically distributed errors with zero means and constant variances. Without loss of generality, we let \( k = 2 \), and \( D_t = t \), and rewrite (1) as

\[\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \alpha \beta' z_{t-1} + \mu + \Psi t + \varepsilon_t,\]

\[D_t = t,\]

The discussion in this section relies heavily on Hansen and Juselius (1995).
\[
\Delta z_t = \Gamma_1 \Delta z_{t-1} + \alpha \beta' z_{t-1} + \mu + \delta t + \epsilon_t,
\]
where \( t \) is a linear time trend. Following Hansen and Juselius (1995), we decompose \( \delta \) and \( \mu \) into
\[
\delta = \alpha \delta_1 + \alpha_0 \delta_2;
\]
\[
\mu = \alpha \mu_1 + \alpha_0 \mu_2,
\]
where \( \delta_1 \) is a \( r \)-dimensional vector of linear trend coefficients in the cointegrating relationship; \( \delta_2 \) is a \((p - r)\)-dimensional vector of quadratic trend coefficients in the data; \( \mu_1 \) is a \( r \)-dimensional vector of intercepts in the cointegrating relationship; and \( \mu_2 \) is a \((p - r)\)-dimensional vector of linear trend slope coefficients in the data.

Substituting Eqs. (3) and (4) into (2), we obtain
\[
\Delta z_t = \Gamma_1 \Delta z_{t-1} + \alpha \begin{pmatrix} \beta' \\ \mu_1 \\ \mu_2 \\ \delta_1 \end{pmatrix} z_{t-1} + \alpha_0 \mu_2 + \alpha_0 \delta_2 t + \epsilon_t,
\]
where \( z_{t-1}' = (z_{t-1}', 1, t) \). Depending on the restrictions on \( \delta_1, \delta_2, \mu_1, \) and \( \mu_2 \), the deterministic components can be modeled in five different ways which we have summarized in Table 1 starting from the most restrictive (Case 1) to the least restrictive (Case 5).

Case 1 does not allow for any deterministic components in the data. This is rather unusual and should only be used with great caution. In Case 2, the model does not allow for any linear trends in the data, but allows for constants in the co-integration space. This is the minimum deterministic component recommended by Johansen (1995) and Hansen and Juselius (1995) since the constants can account for differences in measurement units. In addition to allowing for constants in the co-integration space, Case 3 also allows for linear trends in the data. It is assumed, however, that there are no trends in the co-integration space. Thus, the linear trends enter the VECM as drift (constant) terms. In Case 4, both constants and linear trends are allowed in the co-integration space, in addition to the drift terms. The linear trends in the co-integration space allow for \( r \) trend-stationary stochastic variables, that is, the variables are stationary after detrending, and \( p - r \) variables that are integrated of order one, that is, I(1) processes that also have a linear trend. Thus, this is the model to use if some of the stochastic variables are trend-stationary. Finally, Case 5 places no restrictions on the deterministic components. This case allows for linear trends in the differenced series, \( \Delta z_t \), and hence quadratic trends in \( z_t \). This case is also

| Case 1 | \( \delta_1 = \delta_2 = \mu_1 = \mu_2 = 0 \) |
| Case 2 | \( \delta_1 = \delta_2 = \mu_1 = 0, \mu_2 \neq 0 \) |
| Case 3 | \( \delta_1 = \delta_2 = 0, \mu_1 \neq 0, \mu_2 \neq 0 \) |
| Case 4 | \( \delta_1 = 0, \delta_2 \neq 0, \mu_1 \neq 0, \mu_2 \neq 0 \) |
| Case 5 | \( \delta_1 \neq 0, \delta_2 \neq 0, \mu_1 \neq 0, \mu_2 \neq 0 \) |
considered to be unusual by Hansen and Juselius (1995). They believe that this may be a case of model mis-specification and recommend that more variables be added to the model to increase the information content and to account for the quadratic trends.

It should be clear from the above short review that for the discussion about ‘constants’ or ‘trends’ in Johansen’s co-integration models to be meaningful, they must be related to the co-integration space or excluded from it. It is our contention that this is rarely done, and hence the consequences of mis-specifying the deterministic components in empirical studies are not well known. Perhaps a contributing factor is also that there are no obvious ways to determine which of the deterministic components should be included and whether or not they should be included in the co-integration space.

3. US money demand equations and the data set

There is considerable agreement that money demand is related to an opportunity cost and a scale variable, and we will take this as our maintained hypothesis. We use two monetary aggregates, real M1 and real M2, obtained by deflating the nominal M1 and the nominal M2 by the GDP deflator where the base year is 1992 = 100. For the scale variable, we use real GDP in constant 1992 dollars. To test the robustness of our results, we use three interest rate variables to represent opportunity costs, in addition to the two real monetary aggregates. Two are short-term interest rates and the third is a long-term bond rate. The two short-term interest rates are the three-month Treasury-bill rate and the three-month commercial paper rate. The long-term bond rate is a composite of US Treasury bonds with at least ten years to maturity. We transform all our variables by taking the natural logarithm. In addition, each interest rate variable is included, alternatively, in both the natural logarithm and in the level forms. The source of our data is the Citibase. All data are quarterly from 1959:i to 1998:i except the commercial paper interest rate data which are only available for 1971:ii–1998:i. Note, however, that the effective sample sizes

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6 It should also be clear that Johansen has intended that modeling the deterministic components be an integral part of the model building process. Thus the frequent practice of removing the mean and/or detrending the data prior to conducting co-integration analysis using Johansen’s procedure is inappropriate.

7 Some researchers, for example, Hoffman et al. (1995); and Carlson et al. (2000) also include binary variables to account for structural breaks in their time series. We have not done this for two reasons. First, the published tables of critical values are not applicable when deterministic binary variables are included in the models. Second, we have experimented with including binary variables but did not obtain sensible results, especially with the real M2.

8 Preliminary data analysis suggests that the nominal M2, the GDP deflator, and the nominal GDP appear to be I(2) processes. The real M2 and the real GDP, however, appear to be I(1) processes. Similar results are also reported by King et al. (1991), and Stock and Watson (1993). These results and detailed results for Table 6 are available as Appendix A by request.

9 Quarterly data are averages of monthly data except for GDP and the GDP deflator which are available only quarterly. The computer programs that we used are Rats vs. 4.30 and Cats in Rats.
depend on the number of lags used in the VECM. Finally, all data are seasonally adjusted except for the interest rates, which show no seasonal pattern.

4. Preliminary data analysis

We start our empirical analysis by examining the stochastic properties of the time series. This will allow us to determine whether the time series are trend-stationary series or integrated processes. If the series are integrated processes we need also to determine the order of integration. Our results are reported in Table 2. We first report the Kwiatkowski et al. (KPSS 1992) test for trend stationary series and report the test statistic at lag eight only since the KPSS test statistics decline monotonically with additional lags. The results suggest that we can reject uniformly the null-hypothesis of trend-stationarity at the 5% significance level. The next two columns report the augmented Dickey and Fuller test for one unit root, and two unit roots, respectively. The number in parentheses is the optimal lag length, determined by Akaike’s information criterion. The results show that all the time series are I(1) processes.

Table 2
The stochastic properties of the time series

<table>
<thead>
<tr>
<th>Variables</th>
<th>KPSS test statistic at lag 8</th>
<th>ADF test for one unit root</th>
<th>ADF test for two unit roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>The natural logarithm of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Real M1</td>
<td>0.30</td>
<td>-1.67 (8)</td>
<td>-4.40 (7)*</td>
</tr>
<tr>
<td>2 Real M2</td>
<td>0.30</td>
<td>-1.89 (8)</td>
<td>-4.36 (7)*</td>
</tr>
<tr>
<td>3 Real income</td>
<td>0.32</td>
<td>-2.40 (2)</td>
<td>-6.46 (1)*</td>
</tr>
<tr>
<td>4 Three-month T-bill rate</td>
<td>0.32</td>
<td>-2.34 (3)</td>
<td>-4.92 (8)*</td>
</tr>
<tr>
<td>5 Long-term bond rate</td>
<td>0.37</td>
<td>-0.83 (1)</td>
<td>-9.61 (0)*</td>
</tr>
<tr>
<td>6 Three-month commercial paper rate</td>
<td>0.19</td>
<td>-2.43 (7)</td>
<td>-4.37 (6)*</td>
</tr>
<tr>
<td>The level of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 Three-month T-bill rate</td>
<td>0.29</td>
<td>-2.01 (7)</td>
<td>-5.45 (6)*</td>
</tr>
<tr>
<td>8 Long-term bond rate</td>
<td>0.33</td>
<td>-1.20 (1)</td>
<td>-9.61 (0)*</td>
</tr>
<tr>
<td>9 Three-month commercial paper rate</td>
<td>0.18</td>
<td>-3.36 (5)</td>
<td>-4.14 (6)*</td>
</tr>
</tbody>
</table>

Note: The critical values at the 5% and the 10% significant levels are 0.146 and 0.119, respectively. (*) denotes the rejection of the null hypothesis at the 5% significance level. The critical value for the t-statistic for one unit root at the 5% significance level is -3.43. The critical value for the t-statistic for two unit roots at the 5% significance level is -2.88. The source is Fuller (1976). (k) is the optimal lag length chosen by Akaike’s information criterion.

10 The test results are not much affected if a lag length of twelve is used instead. Note that we present no formal tests of stationarity for these series. A plot of the time series and previous studies of other researchers using the same series suggest that these series are nonstationary.
11 It should be noted that a recent study by Wu and Zhang (1996) reports a finding that short-term interest rates are stationary processes. Johansen (1995) points out that it is a common misconception that all the variables in a co-integration equation must be integrated of the same order. He pointed out that both stationary variables and trend-stationary variables are allowed, provided that there are at least two non-stationary variables that are integrated of the same order.
As we mentioned earlier, the appropriate deterministic components to include in a co-integration relationship are often difficult to determine. We present in Table 3, however, our test results for the deterministic components of the series. The main purpose here is to eliminate from consideration the most unlikely models of the deterministic components, not to identify the most appropriate model of the deterministic components. The most likely model of the deterministic components will be determined jointly with the co-integration rank in the next section using a procedure suggested in Johansen (1992). Thus, the results in Table 3 will also provide the necessary empirical support for the preferred models determined with Johansen’s (1992) procedure. The regression used is

$$\Delta x_t = a_0 + a_1 x_{t-1} + \gamma (\text{trend}) + \sum_{i=1}^{k} \phi_i \Delta x_{t-i} + e_t; \quad (6)$$

where $x_t$ is an individual time series, (trend) is a linear deterministic time trend, and $e_t$ is a serially uncorrelated error term with zero mean and constant variance. In column 1 of Table 3, we report the optimal value of $k$ which is also determined by the Akaike’s information criterion. Column 2 reports the log-likelihood ratio test for the joint hypothesis of a unit root and no deterministic linear trend. This joint hypothesis is not rejected at the 5% or the 10% significance levels in all cases. Assuming that the time series has a unit root, in column 3, we report the $t$-test results for no linear deterministic time trend. This hypothesis is not rejected at the 5% and the 10% significance levels. We report the $t$-test results for a unit root in column 4, assuming that there is no linear deterministic trend. Again, the hypothesis is not rejected at the 5% and the 10% significance levels. The results from columns 2 to 4, together with the KPSS test results reported in Table 2, lead us to conclude that each time series has a unit root and no linear deterministic trend. Thus, Cases 4 and 5 of the models of deterministic components are highly unlikely. The last two columns of Table 3 report the test results for a constant (drift) term assuming that there is no linear deterministic time trend. Column 5 reports the log likelihood ratio test results for the joint hypothesis of a unit root and no drift term. This joint hypothesis is rejected at the 5% significance level for the real M2 and the real income, and at the 10% significance level for the level of the three-month commercial paper rate. Finally, assuming a unit root and no linear deterministic time trend, column 6 reports the $t$-test results for a drift term. This time, the hypothesis is rejected at the 5% significance level for the real M2 and the real income. Thus, our results suggest that the real M2 and the real income contain linear trend, represented by the drift term. The results of columns 5 and 6 suggest that Case 1 of the model of deterministic components is also highly unlikely. This leaves us with Cases 2 and 3 as the two most likely cases. Interestingly, Johansen (1995) and Hansen and Juselius (1995) consider Cases 1 and 5 to be highly unlikely, and Case 4 to be unlikely in actual applications, leaving Cases 2 and 3 to be the most likely cases in actual applications. Moreover, Cases 2 and 3 are the two cases that are found most frequently in the empirical literature on money demand. Of the two cases, Cases 2 and 3, we take no a priori position on the most likely case. Rather, we will present our co-integration results using the deterministic
Table 3
Tests of deterministic components (model: $\Delta x_t = a_0 + a_1 x_{t-1} + \gamma (\text{trend}) + \sum_{i=1}^{k} \phi_i \Delta x_{t-i} + e_t$)

<table>
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<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td></td>
<td>$k$</td>
<td>$H_0: a_1 = \gamma = 0$</td>
<td>$H_0: \gamma = 0$</td>
<td>$H_1: a_1 = 0$</td>
<td>$H_0: a_0 = a_1 = 0$</td>
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</tr>
<tr>
<td>1 Real M1</td>
<td>8</td>
<td>0.46</td>
<td>-0.41</td>
<td>1.44</td>
<td>1.65</td>
<td></td>
</tr>
<tr>
<td>2 Real M2</td>
<td>8</td>
<td>-2.36</td>
<td>-2.78</td>
<td>8.73*</td>
<td>3.05*</td>
<td></td>
</tr>
<tr>
<td>3 Real income</td>
<td>2</td>
<td>-1.15</td>
<td>-1.45</td>
<td>12.18*</td>
<td>4.70*</td>
<td></td>
</tr>
<tr>
<td>4 Three-month T-bill rate</td>
<td>3</td>
<td>-0.25</td>
<td>-2.26</td>
<td>2.56</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>5 Long-term bond rate</td>
<td>1</td>
<td>-1.46</td>
<td>-1.62</td>
<td>1.39</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>6 Three-month commercial paper rate</td>
<td>7</td>
<td>-0.45</td>
<td>-1.93</td>
<td>1.88</td>
<td>-0.18</td>
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The level of

<table>
<thead>
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<th>Variables</th>
<th>(7)</th>
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<td>$H_0: a_1 = \gamma = 0$</td>
<td>$H_0: \gamma = 0$</td>
<td>$H_0: a_0 = 0$</td>
</tr>
<tr>
<td>7 Three-month T-bill rate</td>
<td>7</td>
<td>-0.92</td>
<td>-2.22</td>
</tr>
<tr>
<td>8 Long-term bond rate</td>
<td>1</td>
<td>-1.14</td>
<td>-1.65</td>
</tr>
<tr>
<td>9 Three-month commercial paper rate</td>
<td>5</td>
<td>-0.62</td>
<td>-2.86</td>
</tr>
</tbody>
</table>

Note: (+), (++) denote the rejection of the null hypothesis at the 5% and 10% significance levels, respectively. The optimal lag length determined by the Akaike’s final prediction criterion is given in column 1. The critical values for the $t$-statistics for the results in column 4 are from Fuller (1976), Table 8.5.2. The critical values for the results in columns 2, 3, 5, and 6 are from Dickey and Fuller (1981), Tables 1, 3, 4, and 6, respectively.
components given by Cases 2 and 3. The determination of the most likely case will be left to the procedure discussed in Johansen (1992).

5. Co-integration results

We report the Johansen’s co-integration test results in Tables 4 and 5 for the real M1 and the real M2, respectively. Models A and B in Tables 4 and 5 correspond to Cases 2 and 3, respectively, of the deterministic components discussed in Table 1. There are two test statistics for co-integration, the Lambda-max and the trace statistics. The Lambda-max test statistic tests the null hypothesis of $r$ cointegrating vector(s) against the specific alternative of $r + 1$ cointegrating vector(s). The trace statistic, on the other hand, tests the null hypothesis of no cointegrating vector ($r = 0$) against a general alternative of one or more cointegrating vectors ($r > 0$). We report only the trace statistic in our co-integration tests since we are interested only in whether or not the real M1 and the real M2 are co-integrated with their determinants. We also correct our trace statistics for small sample bias as suggested by Cheung and Lai (1993), and we use the critical values from Hansen and Juselius (1995) for the 5% and 10% significance levels. Finally, the lag lengths used in the VECMs are determined by the shortest lag length that produces serially uncorrelated residuals.

In an earlier version of this paper, we have also considered Case 4 of the deterministic components. A referee, however, points out that Case 4 is not nested in Cases 2 and 3 and suggests dropping Case 4 from consideration. That Case 4 is not nested in Cases 2 and 3 is not essential nor makes any difference in our results for the purpose of demonstrating our main point that different deterministic components can lead to different empirical results. It does produce some minor differences in results when the preferred model of the deterministic components is jointly determined with the co-integration rank using the procedure discussed in Johansen (1992). We will point out the differences in footnote 17 below.

It should be noted that a recent paper by Lütkepohl and Saikkonen (2000) suggest a more efficient estimation procedure for Case 4 of the deterministic components if the linear trend can be assumed to be at most linear and not quadratic.

The correction factor is $(T - nk)/T$, where $T$ is the number of observations, $n$ is the number of variables in the system, and $k$ is the number of lags used.

Stock and Watson (1993) have shown that Johansen’s co-integration test is sensitive to the lag lengths used in the VAR models. Some authors, e.g., Miyao (1996) and Swanson (1998) report results for different lag lengths. We find this procedure to be rather unsatisfactory, since there is no way of knowing whether or not these lag structures are adequate in producing serially uncorrelated error processes. We also want to avoid the inclusion of redundant lags since the Cheung and Lai (1993) correction exacts a heavy toll on those models that have long lags and thus imposes some discipline on the lag selection process. Our purpose here is not to find the correct lag length, which is unknown, but a more modest goal of finding the minimum lag length that would produce serial uncorrelated residuals. In particular, we use two Lagrange multiplier tests for residual autocorrelation. The first is for first-order autocorrelation and the second one is for fourth-order autocorrelation. Both tests are distributed as chi-squared, and a 5% significant level is used. For each interest rate specification, we start with the more restrictive model, Model A of the deterministic components in our paper, and start with one lag, then two, and so on. We soon discover that the fourth-order autocorrelation condition is difficult to satisfy without a minimum of four lags, suggesting to us that there may be residual seasonality in the quarterly data. Once a lag length is determined for Model A, we repeat the same procedure to find the lag length of Model B of the deterministic components.
For Model A, Tables 4 and 5 show that, for every co-integration relationship tested, there is at least one cointegrating vector using the critical value for the trace
statistic at the 5% significance level. This is true regardless of the monetary aggregates used, or the interest rates used, or whether or not the interest rate is in the natural logarithm form or in the level form. Moreover, in four of the twelve cases, there are two cointegrating vectors. Our results, therefore, appear to be fairly robust with respect to different specifications of the co-integration models.

For Model B, the results in Tables 4 and 5 show that of the twelve cases reported, there is not a single case where the null hypothesis of no co-integration is rejected at the 5% significance level. The null hypothesis of no co-integration can be rejected at the 10% significance level in four of the twelve cases. The results, therefore, represent a stark contrast to the results of Model A.

The above discussions show the rather dramatically different results that could be obtained from seemingly similar models, differing only in their deterministic components, and demonstrate the need for a much more careful treatment of the ‘constants’ and ‘trends’ in co-integration models than had been the case in the literature thus far. In the case of the US money demand, the consequences of model mis-specification may also potentially have important consequences for policy recommendations. For example, researchers working with Model A may recommend the use of monetary aggregate targeting as a viable long-run policy, while those working with Model B may reject its usefulness. Given the importance of the money demand equation in policy discussion, we next use a procedure discussed in Johansen (1992) to determine jointly the co-integration rank and the deterministic components of the US money demand models.

The procedure discussed in Johansen (1992) is based on the so-called Pantula (1989) principle. It involves testing sequentially a series of joint hypotheses. Let $T_{rm}$ denote the trace statistic of a model with rank $r$ and deterministic components given by Model $m$. For each specification of the US money demand equation, we start with the trace statistic for $r = 0$, and the most restrictive deterministic components. In our case, this will be $T_{0A}$. If this model is rejected by the critical values (5% or 10%) from the appropriate tables of the trace statistic, we continue to the next model by maintaining the assumption of $r = 0$, but change the model of the deterministic components to the next most restrictive case, that is, $T_{0B}$, in our case. If this joint hypothesis is rejected, we continue next to the trace statistic $T_{1A}$, then to $T_{1B}$, etc. This process continues until the preferred model is identified by the first time that the joint hypothesis is not rejected.

Turning first to the real M1 results in Table 4. Using the procedure described above, the preferred money demand models identified using the 5% significance level in all cases have co-integration rank of zero and deterministic components given by Model B. These results are robust regardless of how the interest rate is defined. The results are only slightly different if the 10% significance level is used instead. We find

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16 We suggest the use of the Pantula (1989) principle as a simple and practical way to simultaneously determine the co-integration rank and the deterministic components of a co-integration model. There are, of course, other procedures that a researcher may choose to use. As one referee points out, a researcher may also analyze the preponderance of evidence, obtained using different samples and lag lengths, based on a model of deterministic components that are reasonable in the context of the application.
one preferred model with one co-integration rank and deterministic components given by Model A. The remaining five preferred models all have co-integration rank of zero and deterministic components given by Model B. The results suggest strongly that there is no equilibrium long-run relationship between the US real M1 and its determinants.

The real M2 results in Table 5 are similar to those of the real M1. At the 5% significance level, all the preferred money demand models identified have co-integration rank of zero and deterministic components given by Model B. At the 10% significance level, all six preferred models have deterministic components given by Model B, and three models now have one co-integration rank each. Thus, using the real M2, there is some weak evidence of a long-run equilibrium relationship between the US real M2 and its determinants.

The above procedure for jointly determining the co-integration rank and the deterministic components of the US real M1 and the real M2 money demand equations strongly suggest that Model B is the preferred model of the deterministic components. Moreover, Model B also appears to be supported by the empirical evidence presented in Table 3, thus reducing the possibility of spurious results from model mis-specification.

Finally, we present in Table 6 some rolling-sample results to assess the effect of changing sample sizes on our results. We start with the sample period ending in 1990:iv, then 1991:iv, 1992:iv, and so on with the last sample period ending in 1996:iv, maintaining the same starting date in all cases. The first sample ending period of 1990:iv is chosen for two reasons. First, we want to be sure that the sample size is sufficiently large in order to obtain meaningful results. Second, a few authors, for example, Hoffman et al. (1995), and Miyao (1996), find that the US money demand equations appear to behave differently pre- and post-1990s. Instead of reporting all the co-integration results, we provide only summary information in Table 6. Note that we have also included the full sample results in the last row of Table 6 for comparison purposes. The first thing to note from Table 6 is that, for the smaller sample sizes, it is difficult to find co-integration. Thus, care should be exercised when interpreting the results. Nevertheless, it is clear from the results that there are more cases of co-integration with Model A than with Model B for both the real M1 and the real M2. For the real M1, the evidence of co-integration with the preferred models of the deterministic components found by the Pantula principle is extremely weak. For the real M2, the evidence of co-integration with the preferred models of the deterministic components appears to be stronger, but still rather weak overall, but the evidence appears to favor Model B as the preferred model of the deterministic components when co-integration is found. In sum, we find little in the rolling-sample results to contradict our earlier results.

\[\text{17 As mentioned in footnote 12 above, we have also considered Case 4 of the deterministic components, which we have labeled Model C, in an earlier version of this paper. Without Case 4, we now have one preferred model given by Model A with one co-integration rank at the 10% significance level for the real M1. For the real M2, three instead of two preferred models now have one co-integration rank at the 10% significance level, with deterministic components given by Model B.}\]
In modeling long-run economic relationships, researchers tend to concentrate on the economic relationship and relatively little attention is paid to modeling the deterministic components of the models. Johansen (1995), and Hansen and Juselius (1995) have emphasized that the choice of the deterministic components has important implications for the asymptotic distributions of the test statistics in Johansen’s co-integration tests. Yet, in empirical application, little is known about the consequences of mis-specifying the deterministic components in co-integration models estimated with Johansen’s approach.

We provide such a comparative study using the US real money demand equation as our example. We choose the US real money demand equation both because of its

<table>
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<th>Number of preferred models with at least one cointegrating vector</th>
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<td>M1 5% 5% &amp; 10% M2 5% 5% &amp; 10%</td>
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<tr>
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Table 6
Rolling sample results

Significance levels: 5% and 10%.

6. Summary and conclusions

In modeling long-run economic relationships, researchers tend to concentrate on the economic relationship and relatively little attention is paid to modeling the deterministic components of the models. Johansen (1995), and Hansen and Juselius (1995) have emphasized that the choice of the deterministic components has important implications for the asymptotic distributions of the test statistics in Johansen’s co-integration tests. Yet, in empirical application, little is known about the consequences of mis-specifying the deterministic components in co-integration models estimated with Johansen’s approach.

We provide such a comparative study using the US real money demand equation as our example. We choose the US real money demand equation both because of its
policy importance and because the recent empirical evidence on the US money demand is decidedly mixed. Our study shows that the inclusion or the exclusion of a linear deterministic time trend can produce rather startlingly different results. In particular, we find that when we exclude a linear deterministic time trend from the co-integration models, we obtain rather strong and robust results in favor of the co-integration of the US real M1 and the real M2 with their determinants. On the other hand, when we include a linear deterministic time trend (the drift term) in the VECM but excluded it from the co-integration space, we find generally rather unfavorable co-integration results. It should also be clear that such dramatically different results may potentially lead to rather different policy recommendations. Researchers who find co-integration of the US real M1 and the real M2 may conclude that the M1 and the M2 monetary aggregates are still useful as long-run intermediate targets of monetary policy. Researchers who have included a linear deterministic time trend in their co-integration models may very well arrive at the opposite policy recommendation.

In order to study further the usefulness of using monetary aggregates as intermediate targets, we use a procedure discussed in Johansen (1992) to jointly determine the co-integration rank and the deterministic components of the US money demand model. The preferred models identified through this procedure generally suggest the inclusion of at least a drift term in the VECM, which is consistent with our investigation of the deterministic components of the time series. On the other hand, the evidence in favor of co-integration is extremely weak, however, suggesting that the usefulness of monetary aggregates as long-run intermediate targets may be limited.

In summary, since it is possible to get rather dramatically different empirical results in empirical applications, we recommend strongly that greater attention and care should be devoted to the initial stages of model specification concerning the trend component of a co-integration model than has been generally the case thus far.

Acknowledgements

I wish to thank Stephen Miller for comments on an earlier draft of this paper. In addition, I want to also acknowledge the useful comments from three anonymous referees of this journal. All remaining errors are mine.

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